Statistical Theory of Transient Processes in a Backward-Wave Oscillator with Crossed Fields

S. P. KUZNETSOV

A previously developed nonstationary theory is used to investigate the transient process which leads to establishment of oscillations in a magnetron-type backward-wave tube (MBWT) oscillator from its beginning to its end, including the generation of a regular signal by shot noise and the final nonlinear stage. It is shown that weak nonlinear effects play the principal role in self-excitation of the oscillator when the beam current is close to its starting value. In particular, the fluctuations can switch the system from a non-excited state to an oscillatory state when the beam current is smaller than its starting value. A method for calculation of the mean value and the variance of the length of transient process is proposed and illustrated by examples.

* * *

INTRODUCTION

The purpose of this article is to investigate the process of beginning of oscillation and of appearance of a regular signal from noise in a backward-wave oscillator with crossed fields (MBWT), using the previously developed nonstationary, nonlinear theory [1-3] as a base. This problem has some practical significance and is also of a theoretical interest as an example of the effect of fluctuations on a specific, distributed, self-oscillatory system. This problem has been solved previously in the form of estimates [4, 5], but the authors of these works did not consider a number of important aspects (they did not investigate the role of the nonlinear effects and of the method used to turn on the oscillator in the transient process and they did not estimate the average dispersion of the length of the transient process).

In what follows below we use the physical model of the MBWT described in detail in references 1-3 . We shall only point out that we will use the adiabatic approximation, that the effects of space charge field, of the distributed attenuation, of reflections from the ends of the tube and of the non-constant group velocity in the spectral interval of the signal are neglected and that the gain parameter D is assumed to be small ($D \ll 1$). It is also assumed that the fluctuations are caused by the shot effect and that the electrons enter the interaction space independently from one another, being randomly distributed in terms of the time of entry and over the transverse cross section of the beam.

1. ESTABLISHMENT OF OSCILLATIONS FROM NOISE DURING A STATIONARY ELECTRICAL OPERATING MODE OF THE TUBE

Let us consider the simplest way in which the oscillations in a MBWT can be generated by noise, i.e., when the process takes place under constant values of electrode voltages and beam current. We shall assume that the shot noise was "turned off" prior to the time $t = 0$ and that the oscillator was in an unexcited state.*

1) If the electron beam current I_0 is larger than the so-called starting current I_1 then the unexcited state of the oscillator is unstable with respect to infinitesimally small oscillations at a frequency ω which is determined by equating the phase velocity of the wave propagating in the transmission line to the velocity of motion of the electrons [4-6]:

$$v_{\text{ph}}(\omega) = v_0. \tag{1}$$

Since the spectrum of noise fluctuations always contains components at frequencies that are close to ω, these components will increase as the instability develops, causing the appearance of a regular signal. The qualitative picture of this process corresponds to that given in reference 6 , pages 340 and 185.

2) If the beam current is smaller than I_1 but larger than some value I_2, then the unexcited state of the oscillator is unstable at the same frequency with respect to perturbations of finite amplitude

*Even though such a state is unrealistic, it can sometimes be considered as a wise idealization of the physical process. Moreover, it will serve as a starting point for analysis of more complicated cases (see Section 2).

ISSN0033-7889/78/0030-0074$07.50/0

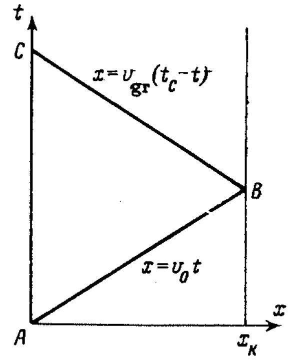

Fig. 1. Backward radiation of an electron moving on the trajectory AB occupies the region ABC on the space-time diagram.

(which becomes smaller as the beam current approaches its starting value), see [7, 1-3]. This is related to the specific character of the weak nonlinearity existing in the MBWT: the larger the amplitude of the signal the more significant becomes the displacement of the electrons contained in the decelerating phase of the wave with respect to the retarding system. There they fall into an always stronger and stronger high-frequency field and give up their potential energy to the wave more and more effectively.

In that case the fluctuations can also lead to self-excitation, but its mechanism is somewhat different. Since the system amplifies predominantly the noise components at frequencies that are close to the operating frequency, we can assign the frequency ω to the output noise signal and assume that its amplitude and phase are slowly varying functions of time. If at some instant of time the amplitude exceeds a specified level, then it will start to increase fast, according to our discussion above. In a strongly nonlinear mode this growth of the amplitude is limited by the precipitation of electrons onto the retarding system. The average time required for self-excitation of the oscillator will obviously become shorter as I_0 approaches closer to I_1.

3) If $I_0 < I_2$ then the oscillator cannot become excited by itself in principle. For sufficiently long values of t the noise at the output of the tube becomes a stationary random process. This is the so-called pre-oscillatory mode of the tube.

1.1. Derivation of MBWT responses to an "impulse" caused by a separate electron. Let us consider an electron which gives rise to the current density

$$\vec{j}=e\vec{v}_0\delta(x-v_0t)\,\delta(y-\Delta y)\,\delta(z), \tag{2}$$

and whose trajectory is depicted by the line AB on the space-time diagram (Fig. 1). Using the results of reference 8, it is possible to show that its radiation field at a frequency ω which satisfies condition (1) occupies the region ABC (group velocity of the wave is negative) and is described by the expressions:

$$\vec{E}=\mathrm{Re}\,[C_{-s}\vec{E}_{-s}{}^0e^{i(\omega t-\beta x)}], \tag{3}$$

$$C_{-s}=\frac{2eE_{sx}{}^0(\Delta y)}{N_s(v_0{}^{-1}+v_{gr}{}^{-1})}\simeq\frac{2e[E_{sx}{}^0(0)-i\beta\Delta yE_{sy}{}^0(0)]}{N_s(v_0{}^{-1}+v_{gr}{}^{-1})}, \tag{4}$$

where $\vec{E}_{\pm s}{}^0$ are the eigenfunctions of the waveguide at the frequency ω, N_s is a norm, v_{gr} is the absolute value of the group velocity of the wave and $\beta = \omega/v_0$. We shall now change (4) to the following dimensionless parameters:

$$F=\varepsilon u\frac{1-iw}{1+u}=F_e, \tag{5}$$

where $F = C_{-s}/D$ is the normalized field amplitude, $u = v_{gr}/v_0$ is the normalized group velocity, $w = \beta\Delta y$ and $\varepsilon = e\omega D/I_0$ is a parameter which characterizes the discrete structure of the electron beam and, consequently, the level of the shot noise.

In order to be able to consider the interaction between the electron beam and the radiation described by formulas (3)-(5), we shall use the linearized equations of the nonstationary MBWT theory [1-3]:

$$\frac{1}{u}\frac{\partial F}{\partial \tau}-\frac{\partial F}{\partial q}=\Phi, \qquad \frac{\partial \Phi}{\partial \tau}+\frac{\partial \Phi}{\partial q}=F \tag{6}$$

with boundary conditions

$$\Phi|_{q=0}=0, \quad F|_{q=l}=0 \tag{7}$$

and the following initial condition on the curve AB

$$F|_{q=\tau}=F_e. \tag{8}$$

Here $\tau = \omega Dt$ and $q = \beta Dx$ and Φ is an auxiliary grouping parameter. Solving this problem by using the Laplace transform with respect to time variable, we obtain

$$F(0,\tau)=\frac{F_e}{\pi i}\int_{\chi-i\infty}^{\chi+i\infty}\frac{\exp\left(s\frac{2u\tau}{1+u}\right)}{s+\sqrt{1-s^2}\operatorname{ctg}\sqrt{1-s^2}\,l}\,ds, \tag{9}$$

where χ is selected in such a way that the contour of integration passes to the right of all singularities of the integrand in the plane of complex variable s.

1.2. Establishment of oscillations when the electron beam current is significantly higher than the starting current, $\langle I_0 \geqslant 1.01 I_1\rangle$.* In terms of our normalized parameters the condition $I_0 > I_1$ means that $l>\pi/2$. In that case the integrand of expression (9) has one or several poles in the right-hand half-plane and it turns out that all these poles are located on the real axis [1-3]. Integrating (9) by taking into account only the largest pole at $s = \varkappa$, we obtain the following asymptotic form of the response for large values of τ:

$$F(0,\tau)=2F_e\frac{1-\varkappa^2}{1+\varkappa l}e^{\alpha\tau}, \tag{10}$$

where $\alpha = 2u\varkappa/(1+u)$.

During the time interval ranging from 0 to t an average of $\bar{n} = I_0 t/e = \tau/\varepsilon$ will enter the interaction space, so that the resultant response is a superposition of $\bar{n}$ terms of the type of (10), i.e.,

$$F(0,\tau)=\sum_{n=1}^{\bar{n}}F_n(0,\tau)=\frac{2u\varepsilon}{1+u}\frac{1-\varkappa^2}{1+\varkappa l}\sum_{n=1}^{\bar{n}}(1-iw_n)e^{\alpha(\tau-\tau_n)+i\varphi_n}, \tag{11}$$

where the random quantities τ_n (time of entry), φ_n (phase) and w_n (deviation from the beam axis) pertain to the n-th electron and are henceforth assumed to be uncorrelated.

Further derivation is analogous to the case of lumped-parameter self-oscillatory systems [9, 10] and for this reason we shall present only the final results.

If we write the amplitude of the output signal in the form $|F|=Ae^{\alpha\tau}$ then the random variable A will have the Rayleigh distribution

$$f(A)=\frac{A}{\sigma^2}e^{-A^2/2\sigma^2}, \tag{12}$$

where the variance σ^2 is given by the following expression when $e^{2\alpha\tau}\gg 1$:

$$\sigma^2=\frac{\bar{n}}{2}\langle|F_n|^2\rangle\simeq\frac{\varepsilon u(1+\langle w^2\rangle)}{2\varkappa(1+u)}\left(\frac{1-\varkappa^2}{1+\varkappa l}\right)^2. \tag{13}$$

The distribution function of the time τ_R, required by the amplitude to reach a level $|F| = R$ has

---

*The case $I_0 \simeq I_1$ will be discussed in the next section. Let us point out that the ranges of applicability of approaches developed in Sections 1.2 and 1.3 overlap partially.

the following form

$$f(\tau_R)=\frac{\alpha R^2}{\sigma^2}\exp\left[-2\alpha\tau_R-\frac{R^2}{2\sigma^2}\exp(-2\alpha\tau_R)\right]. \tag{14}$$

A graph of this function can be found in reference 10 .
Using (14), it is not difficult to find the average value of the time τ_R and its RMS deviation:

$$\alpha\langle\tau_R\rangle=\int_0^\infty \alpha\tau f(\tau)\,d\tau \simeq \frac{1}{2}\left(\ln\frac{R^2}{2\sigma^2}+0{,}577\right), \tag{15}$$

$$\alpha\sqrt{\langle(\Delta\tau_R)^2\rangle}=\frac{\pi}{8}. \tag{16}$$

It is wise to use the obtained results in the following way: 1) we select a value of the amplitude R which is large in comparison to the noise level ($R \gg \sigma$) and at the same time sufficiently small for the nonlinear effects to be inconsequential; 2) on the basis of the linear fluctuation theory described above we determine the average time required by the amplitude to reach the level R and the average dispersion of that time; 3) using a digital computer and the nonlinear, nonstationary theory, we determine the variation of amplitude beginning from the level R until the steady state is reached. Here we use the following equations derived in references 1-3 :

$$\frac{1}{u}\frac{\partial F}{\partial \tau}-\frac{\partial F}{\partial q}=G(\Phi), \qquad \frac{\partial \Phi}{\partial \tau}+\frac{\partial \Phi}{\partial q}=F \tag{17}$$

with boundary conditions (7) and the following initial conditions:

$$\Phi|_{\tau=0}=R\exp\left(\frac{1-u}{1+u}\varkappa q\right)\frac{\sin\sqrt{1-\varkappa^2}\,q}{\sin\sqrt{1-\varkappa^2}\,l}, \tag{18}$$

$$F|_{\tau=0}=R\exp\left(\frac{1-u}{1+u}\varkappa q\right)\frac{\sin\sqrt{1-\varkappa^2}\,(l-q)}{\sin\sqrt{1-\varkappa^2}\,l};$$

The method for solution of these equations was also described in references 1-3 ; 4) using the formula $t = \tau/\omega D$ we convert the results to normal time.

Let us now give an example of calculation of the length of transient process in a specific oscillator (tube parameters were taken from monograph [11]).

1) $I_0 = 1.3 I_1 = 0.13$ amperes, $D = 0.022$, $\omega = 1.9 \cdot 10^{10}\ \text{sec}^{-1}$, $u = 1$, $\varkappa = \alpha = 0.194$ and $\langle w^2\rangle = 0.1$.

a) We determine the parameter $\varepsilon = e\omega D/I_0 \simeq 4.7 \cdot 10^{-10}$.

b) From formula (13) we obtain the quantity $\sigma^2 \simeq 3.4 \cdot 10^{-10}$.

c) Letting $R = 0.2$, we obtain $\langle\tau_R\rangle \simeq 48$ from (15) $\sqrt{\langle(\Delta\tau_R)^2\rangle} \simeq 2$ from (16).

Figure 2a shows the graph of the output signal amplitude as a function of time for the most probable realization. It may be seen from the figure that the substantially nonlinear interval is not less than 20% of the entire transient time. It should also be noticed at the same time that an estimate obtained on the basis of linear theory* gives a satisfactory result (see dashed curve in Fig. 2a).

2) $I_0 = 3.65 I_1 = 0.365$ amperes, $D = 0.031$, $\omega = 1.9 \cdot 10^{10}\ \text{sec}^{-1}$, $u = 1$, $\varkappa = \alpha \simeq 0.65$ and $\langle w^2\rangle = 0{,}1$.

Completing the same sequence of calculations we find $\langle\tau_R\rangle \simeq 19$ and $\sqrt{\langle(\Delta\tau_R)^2\rangle} \simeq 0{,}6$ for $R = 0.2$. The corresponding graph of the transient process is shown in Fig. 2b. Its general shape is the same as in the previous case, but now the time required to reach steady state is substantially shorter.

1.3. Transient process for a beam current which is almost equal to the starting current. In this case one must expect that the effect of weak nonlinearity will become substantial during the initial stage of the process. We let $I_0 = I_1(1+\mu)$, where $\mu \ll 1$ is a dimensionless parameter. It is shown in Appendix 1 that then the nonlinear system of MBWT Eqs. (17), with appropriate boundary conditions, reduces approximately to the following ordinary differential equation:

$$\frac{dC}{d\tau}=\frac{\pi u}{4(1+u)}(2\mu+|C|^2)C,$$

*In order to obtain such an estimate, it is necessary to use relationship (15), in which R is replaced by the steady-state amplitude, obtained from the nonlinear theory.

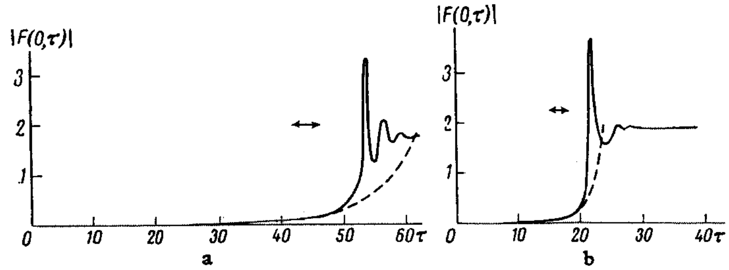

Fig. 2. Curves of the output signal of a MBWT, plotted as functions of time for the most probable realization with $\beta y_{r.s.} = 3$: a) $l = 1.8$; b) $l = 3$. Dashed curves correspond to the linear theory. The arrow indicates the average dispersion of the transient time. For the tube selected [11] one unit of normalized time corresponds to 2.4 nanoseconds in the first case and to 1.7 nanoseconds in the second case.

where $C = F(0, \tau)$ is the complex amplitude of the signal at the output of the tube. In the absence of fluctuations the solution of Eq. (19) is as follows

$$|C| = \sqrt{2\mu}\left[\left(1+\frac{2\mu}{|C_0|^2}\right)\exp\left(-\mu\frac{\pi u\tau}{1+u}\right)-1\right]^{-1/2}; \quad \arg C = \text{const}, \tag{20}$$

where $|C_0|$ is the signal amplitude at the time $\tau = 0$. It may be seen from formula (20) that the amplitude will grow to infinity during the finite time interval

$$\tau_\infty = \frac{1+u}{\mu\pi u}\ln(1+2\mu|C_0|^{-2}) \tag{21}$$

provided that a) $\mu > 0$, b) $\mu < 0$ and $|C_0|^2 > 2\mu$ (instability with respect to perturbations of finite amplitude). In a physical tube the amplitude of oscillations is limited by precipitation of electrons onto the retarding system which was not considered in derivation of (19).

Since the problem is being solved under the approximation of a weak nonlinearity, we assume that relationship (10), obtained from the linear theory, can be used in order to determine the result of perturbation of the system by an impulse caused by a single electron. Since $I_0 \simeq I_1$, $\varkappa \simeq 0$ and the pole of the integrand function has the largest real part, we have

$$F(0, \tau) \simeq 2F_e \quad \text{when} \quad \tau \to \infty. \tag{22}$$

Since the time required by one impulse is small in comparison to the typical time of amplitude variation $[l(1 + u^{-1}) \ll (2/\pi\mu)(1 + u^{-1})]$, we can assume that the mapping point in the complex plane, whose motion is described by Eq. (19), experiences on the average $1/\varepsilon$ jumps per unit of normalized time in a random direction, the mean squared length of the jump being equal to $[4\varepsilon^2 u^2/(1 + u^2)](1 + \langle w^2\rangle)$. Then we can write a Fokker-Plank equation whose solution would allow us to investigate the evolution of a statistical ensemble of oscillators in detail [12]. In this article, however, we shall limit ourselves to the derivation of the average time of self-excitation of the oscillator. We shall assume tha this time is the time required by the amplitude to grow to infinity.

Suppose that M(R) is the average time required by the mapping point to reach infinity when it was originally located at a distance R from the origin of coordinates. This function satisfies the equation [12]

$$\frac{b}{2}\left(M'' + \frac{1}{R}M'\right) + \frac{\pi u}{4(1+u)}R(R^2+2\mu)M' = -1 \tag{23}$$

with the boundary conditions

$$M(\infty) = 0, \quad M'(0) = 0, \tag{24}$$

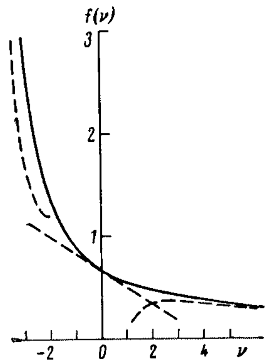

Fig. 3. Graph of function f(ν). Dashed curves correspond to approximations (A.7)-(A.9).

where

$$b = \frac{2\varepsilon u^2}{(1+u)^2}(1+\langle w^2\rangle)$$

is the coefficient of diffusion.

Integrating Eq. (23), we obtain M(0):

$$M(0) = \sqrt{\frac{2(u+1)}{\pi u b}}\, f\left(\mu\sqrt{\frac{2\pi u}{b(u+1)}}\right), \tag{25}$$

where

$$f(\nu) = \int_0^\infty e^{-x^2-\nu x}\frac{dx}{x}\int_0^x e^{\xi^2+\nu\xi}\, d\xi. \tag{26}$$

Approximate expressions for the function f (ν) are derived in Appendix 2 and its graph is shown in Fig. 3.

Let us now assess the obtained results, using the tube with the previously described parameters as an illustrative example.

a) $\mu < 0$. In this case the self-excitation of the oscillator can be treated as a "flipover" from one state to another, caused by fluctuations. It may be seen from formula (A.7) (Appendix 2) that the waiting time for the flipover depends very strongly on μ (since $\varepsilon \sim 10^{-9}$ and $\nu \sim \mu/\sqrt{\varepsilon}$ appears as a square in the exponent). A calculation based on (A.7) gives

μ	$M(0)$	$M(0)$, sec
$0{,}8\cdot 10^{-3}$	$\sim 8\cdot 10^{4}$	$\sim 2{,}8\cdot 10^{-4}$
10^{-3}	$\sim 3\cdot 10^{9}$	~ 10
$1{,}2\cdot 10^{-3}$	$\sim 2\cdot 10^{15}$	$\sim 6\cdot 10^{6}$
$1{,}4\cdot 10^{-3}$	$\sim 10^{22}$	$\sim 10^{14}$

i.e., the self-excitation of the oscillator by the above described mechanism becomes very unlikely already when $\mu \gtrsim 1.2 \cdot 10^{-3}$. Therefore, it is actually not necessary to investigate the case $I_2 < I_0 < I_1$ on the basis of Eqs. (17).

b) $\mu = 0$. A calculation based on formula (A.8) results in $M(0) \simeq 5 \cdot 10^4$ or $\sim 1.7 \cdot 10^{-4}$ seconds.

c) $1 \gg \mu \gg \sqrt{\varepsilon}$. In this case relationship (A.9) becomes valid, which agrees with the results obtained in Section 1.2.

1.4. Pre-oscillatory mode.* Since in this case the noise signal at the tube output remains small in principle, we shall use a linear approximation. The integrand function in formula (9) has now no

*The approach discussed below will also be used when $I_2 < I_0 < I_1$ if the observation time interval is small in comparison to the time required to excite the oscillator.

Condition imposed on T_0	Condition imposed on I_0	Character of the process	Method of analysis
$\omega T_0 \leqslant 1$	$I_0 > I_1$	Transient process is started by the radiation of the electron beam front;	We determine the spectral component of the front radiation at the frequency ω. The result is used as an initial condition for Eq. (6) or (17)
	$I_1 > I_0 > I_2$	If the radiation of the front exceeds the level of instability, the oscillator becomes self-excited due to a weak nonlinearity.	Same. When $I_0 \simeq I_1$ one can use (20), where $\vert C_0 \vert$ is the amplitude of front radiation.
$\omega T_0 \gg 1$ $T_0 \ll T_{tr}$	$I_0 > I_1$	Signal appears from noise in the linear mode when the tube bias voltages are maintained constant.	Formulas (15) and (16) are applicable.
	$I_0 \simeq I_1$	Same, but in a weakly nonlinear mode.	Formula (25) is applicable.
	$I_1 > I_0 > I_2$	When the beam current is far from the starting value, the probability of excitation is very small.	
$T_0 \geqslant T_{tr}$	$I_0 > I_1$	Signal appears from noise in the linear mode. The increment varies substantially during the transient process.	Linear theory must be modified by taking into account $\varkappa(\tau)$.
	$I_0 \simeq I_1$	Same, but in a weakly nonlinear mode.	
$T_0 \gg T_{tr}$	$I_0 > I_1$	As one approaches the starting mode slowly, the device will eventually flip over to oscillations. The most important role is played by the weak nonlinearity.	Nonstationary analysis of the Fokker-Planck equation is required.

singularities in the right-hand half-plane of s and this expression can be changed from Laplace transform to Fourier transform by letting $s = i\Phi_0$. Evaluating the spectral density function of the response and summing this function together over all electrons that have entered the interaction space per unit of time, we obtain the following normalized power spectral density of noise in the pre-oscillatory mode:

$$W = \frac{4\varepsilon u^2(1+\langle w^2\rangle)}{(1+u)^2}\,\frac{\sin^2\sqrt{1+\Phi_0^2}\,l}{\Phi_0^2+\cos^2\sqrt{1+\Phi_0^2}\,l}. \tag{27}$$

An analogous relationship was derived in reference 13 by another method.

2. ON THE RANGES OF APPLICABILITY OF THE DEVELOPED THEORY

Let us now discuss the processes which lead to stable generation when the methods used to turn on the oscillator are more realistic. We shall assume that constant voltages are applied to the retarding system and to the negative electrode and that the beam current increases from zero to some value I_0 during some typical time T_0 according to the following function:

$$I(t) = I_0\varphi\left(\frac{t}{T_0}\right), \quad \varphi \underset{t\to-\infty}{\rightarrow} 0; \quad \varphi \underset{t\to\infty}{\rightarrow} 1. \tag{28}$$

If the beam which enters the interaction space has a distinctive front (whose width is of the order of the retarded wavelength), then the radiation from this front at the frequency ω can be larger than the noise level and will therefore start the transient process. In a frequency interval $\Delta\omega \sim \alpha\omega D$ (α is a normalized increments increase of small oscillations) the energy of the radiation field will be proportional to $|I_\omega|^2(\Delta\omega^2/4\pi^2)$, where $I_\omega = I_0 T_0 \Psi(\omega T_0)$ and $\Psi(\zeta) = \int_{-\infty}^{\infty} \varphi(\xi) e^{-i\xi\zeta} d\xi$. On the other hand, in the same frequency interval the energy of radiation field of the noise current is proportional to $\langle I_{noise}^2 \rangle =$ $= eI_0(\Delta\omega/\pi)$ according to Shottky's formula for the mean-squared current fluctuations. Thus the radiation of the front will dominate above noise if

$$|I_\omega| \gg \sqrt{\frac{4\pi e I_0}{\Delta\omega}} \quad \text{or} \quad \omega T_0 |\Psi(\omega T_0)| \gg \sqrt{\frac{4\pi e \omega}{\alpha D I_0}}, \tag{29}$$

where the typical value of the quantity contained in the right-hand side is $\sim 10^{-3}$. One can maintain with certainty that this inequality will be satisfied when $\omega T_0 \ll 1$. It is much more difficult to draw some definite conclusions when $\omega T_0 \gg 1$. As a matter of fact, the formal asymptotic behavior of the function $\Psi(\omega T_0)$ at large values of ωT_0 is determined by the analytic properties of function φ which are difficult to pinpoint (for example, when φ has a discontinuous derivative of n-th order than $|\Psi| \sim (\omega T_0)^{-n-1}$, when φ is differentiable an infinite number of times at any point then $|\Psi| \sim e^{-\omega T_0}$ or approaches zero even faster). The physical meaning of this difficulty amounts to answering the question: Does the function $\varphi(t/T_0$ have "irregularities" which are unrelated to noise* and have a time scale $T \ll T_0$ or does it not? If it does, then their radiation can dominate above noise even when $\omega T_0 \gg 1$. If we assume that such irregularities do not exist and that $|\Psi(\omega T_0)| \leqslant e^{-\omega T_0}$, then it follows from (29) that steady-state oscillations will be established from noise when $\omega T_0 \geqslant 5$. When this inequality is satisfied, we can use the methods developed in the last section in order to find the transient time T_{tr} under the conditions of constant electric bias. This time will be a good approximation to the total transient time if $T_0 \ll T_{tr}$. Otherwise the theory requires modifications which are beyond the scope of this article. A summary of basic possible transient modes in a MBWT is given in the table above together with brief explanations of the essence of physical processes and of the methods of analysis.

CONCLUSIONS

1. Using a specific problem as an example and making some specific assumptions it was shown that the methods developed in the classical theory of self-oscillators with lumped parameters are applicable to investigation of distributed self-oscillatory systems.
2. The process of excitation of oscillations in a MBWT was examined for the first time from beginning to the end, including its linear and nonlinear stages. It was shown that an estimate of the time required to reach steady-state oscillations, based on methods of references 4, 5, is satisfactory when the beam current is significantly higher than the starting current, i.e., $\langle I_0 \gg 1.01 I_1 \rangle$.
3. It was pointed out that the nonlinearities related to the rise of electrons toward the retarding system have a principal effect on the process of self-excitation of a MBWT when the beam current value is close to the starting current value. A method for calculation of the transient time for this case was proposed.
4. Different possible transient modes of a MBWT were discussed qualitatively.

The author wish to thank D. I. Trubetskov for his attention to this work.

APPENDIX 1

If $I_0 = I_1(1 + \mu)$ then the normalized length of the tube is

$$l \simeq \frac{\pi}{2}\left(1 + \frac{\mu}{2}\right),$$

*In a formal approach such irregularities would be discontinuities of derivatives, for example.

because $l \sim D \sim \sqrt{T_0}$. We now change the variables in Eqs. (17) to $q = q'(1+\mu/2)$, $\tau = \tau'(1+\mu/2)$ and $F = F'(1+\mu/2)$. Dropping the primes, we shall have

$$\frac{\partial F}{\partial q} + \Phi = \frac{\partial F}{\partial \tau} - g(\Phi), \quad \frac{\partial \Phi}{\partial q} - F = -\frac{\partial \Phi}{\partial \tau}, \tag{A.1}$$

$$\Phi(0, \tau) = 0, \quad F\left(\frac{\pi}{2}, \tau\right) = 0, \tag{A.2}$$

where $g(\Phi) = (\mu\Phi + \mu)G(\Phi) - \Phi \simeq 1 - {}^2/{}_3\Phi^3$. This is true for small values of Φ and can be obtained by expanding $G(\Phi)$ [7, 1-3] into a Taylor series. Neglecting the right-hand sides of (A.1), we obtain the following zero-order approximation

$$F^{(0)} = C \cos q, \quad \Phi^{(0)} = C \sin q. \tag{A.3}$$

We now substitute (A.3) into right-hand sides of (A.1), assume that $C = C(\tau)$ and solve the obtained equations for F and Φ. Insisting that the solution should satisfy boundary conditions (A.2), we arrive at the equation

$$\frac{\partial C}{\partial \tau} = \int_0^{\pi/2} g(C \sin q) \sin q \, dq. \tag{A.4}$$

From this we obtain Eq. (19) by using an explicit expression for $g(\Phi)$.

APPENDIX 2

We shall consider the function

$$\Psi(\varphi, \nu) = \int_0^\infty e^{-x^2 - \nu x} \frac{dx}{x} \int_0^{x \cos 2\varphi} e^{\xi^2 + \nu\xi} \, d\xi,$$

where it is obvious that $\Psi(0, \nu) = f(\nu)$ and $\Psi(\pi/4, \nu) = 0$. Then

$$\frac{\partial \Psi(\varphi, \nu)}{\partial \varphi} = -\frac{\sqrt{\pi}}{2} \exp\left(\frac{1}{4}\nu^2 \operatorname{tg}^2 \varphi\right) \operatorname{erfc}\left(\frac{1}{2}\nu \operatorname{tg} \varphi\right). \tag{A.5}$$

Integrating formula (A.5), we obtain the expression

$$f(\nu) = \frac{\sqrt{\pi}}{2} \int_0^{\pi/4} \exp\left(\frac{1}{4}\nu^2 \operatorname{tg}^2 \varphi\right) \operatorname{erfc}\left(\frac{1}{2}\nu \operatorname{tg} \varphi\right) d\varphi, \tag{A.6}$$

which is convenient to use for approximate representations of $f(\nu)$.

1) $\nu < 0$, $|\nu| \gg 1$. In the vicinity of the point $\varphi = \pi/4$, which makes the principal contribution to the integral, we have $\operatorname{erfc}(1/2\, \nu \operatorname{tg} \varphi) \simeq 2$. Therefore we obtain the following by the Laplace method [14]

$$f(\nu) \simeq \frac{\sqrt{\pi}}{\nu^2} \exp \frac{\nu^2}{4}. \tag{A.7}$$

2) $\nu \simeq 0$. We integrate assuming that

$$\exp\left(\frac{\nu^2}{4} \operatorname{tg}^2 \varphi\right) \simeq 1; \quad \operatorname{erfc}\left(\frac{\nu}{2} \operatorname{tg} \varphi\right) \simeq 1 - \frac{\nu}{\sqrt{\pi}} \operatorname{tg} \varphi:$$

$$f(\nu) \simeq \frac{\pi\sqrt{\pi}}{8} - \frac{\nu}{4} \ln 2. \tag{A.8}$$

3) $\nu \gg 1$. In this case it seems to be wise to use the asymptotic formula $\operatorname{erfc} x \sim e^{-x^2}/x\sqrt{\pi}$ for large

values of x. When this expression is substituted into (A.6), however, the integral diverges. Therefore, we let erfc $x \sim e^{-x^2}/(x+\Delta)\sqrt{\pi}$, where $\Delta > 0$ is an arbitrary number. Then the integration gives

$$f(\nu) \simeq \frac{1}{\nu(1+4\Delta^2\nu^{-2})} \left[\frac{\pi\Delta}{2\nu} + \ln\left(\frac{1}{\sqrt{2}} + \frac{\nu}{2\sqrt{2}\Delta}\right)\right] \sim \frac{\ln\nu}{\nu}, \tag{A.9}$$

where terms of the order of $\sim\nu^{-1}$ and smaller have been neglected. Let us point out that a specific choice of Δ is inconsequential because this quantity does appear in the final result.

REFERENCES

1. S. P. Kuznetsov and D. I. Trubetskov, Lektsii po elektronike (3-ya zimnyaya shkola-ceminar inzhenerov) [Lectures in Electronics (3-rd Winter Seminar School for Engineers)], book 5, Saratov University Press, 1974, pp. 88-142.
2. Elektronika lamp s obratnoy volnoy (Elektronics of Backward Wave Tubes), Saratov University Press, 1975, Chapter III.
3. S. P. Kuznetsov and D. I. Trubetskov, Izv. Vuzov MVSSO SSSR (Radiofizika), 1977, 20, No. 2, p. 300.
4. Mourier, in the book: Elektronnyye cverkhvysokochastotnyye pribori so skreshchennymi polymi (Electronic Microwave Devices with Crossed Fields), IL Press, 1961, I, p. 347.
5. D. I. Trubetskov and V. P. Shakhin, in coll. volume: Voprosy elektroniki sverkhvysokikh chastot (Topics in Microwave Electronics), Saratov University Press, 1969, ed. 6, p. 10.
6. V. N. Shevchik and D. I. Trubetskov, Analiticheskiye metody rascheta v elektronike SVCh (Analytic Design Methods in Microwave Electronics), Sovetskoye Radio Press, 1970.
7. Feinstein, Kayno, in the book: Elektronnyye sverkhvysokochastotnyye pribory so skreshchennymi polyami (Electronic Microwave Devices with Crossed Fields), IL Press, 1961, I, p. 451.
8. V. A. Solntsev and A. S. Tager, Radiotekhnika i Elektronika, 1960, 5, No. 7, p. 1100 [Radio Eng. and Electron. Phys., 5, No. 7 (1960)].
9. W. A. Edson, Proc. IRE, 1960, 8, 1454.
10. I. S. Gonorovskiy, Trudy MAI, ed. 50, 1955, p. 125.
11. V. I. Gayduk, K. I. Palatov and D. M. Petrov, Fizicheskiye osnovy elektroniki SVCh (Physical Fundamentals of Microwave Electronics), Sovetskoye Radio Press, 1971.
12. A. A. Andronov, A. A. Vitt and L. S. Pontryagin, ZhETF, 1933, 3, No. 3, p. 165; A. A. Andronov, Sobraniye trudov (Collection of Works), AN SSSR Press, 1956, p. 142.
13. B. Epsztein and B. Glance, Etude du fonctionnement du carcinotron M en rigime de pre-oscillation, Tubes hyperfrequences, Paris, 1965, 183-186.
14. E. Copson, Asymptotic Expansions (in Russian transl.), Mir Press, 1966.

□◆◆◆□