## TOTAL SIGNAL SUPPRESSION REGIMES IN TRAVELING WAVE TUBES OPERATING NEAR THE LIMITS OF THE PASSBAND

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A wave approach is used to study total signal suppression regimes in a traveling wave tube near the limits of the passband. The possibility of several types of suppression regimes is proven. A regime termed reactive suppression is predicted, in which interaction between field and current is purely reactive.

1. As is well known, when beam current and accelerating voltage are properly selected in a traveling wave tube (TWT), a regime producing complete suppression of an input signal of a certain frequency is realized [1]. It is convenient to use this regime for measurement of parameters, since it is totally defined by interaction of the beam and field and is independent of input and output matching conditions. (In this respect the suppression regime is unique among the possible operating regimes of a finite length system.)

The present study will prove the possibility of realization of several types of suppression regimes in a TWT operating close to the limit of the retarding system passband. Since several of these regimes correspond to parameter values at which an infinitely long system is absolutely unstable [2], it is natural to question the possibility of realization of such regimes, i.e., their stability. (We will note that at limiting parameter values the suppression conditions found here may transform to either Kompfner breakoff conditions or reverse wave tube (RWT) start conditions without reflections.) The answer to this question is that for experimental observation of such regimes it may be necessary to choose conditions at the ends of the retarding system such that at the current level corresponding to the suppression regime the tube is still not self-excited.

2. We will approximate the dispersion characteristic of the retarding system near the passband limit by the first terms of a Taylor series:

$$\omega = \omega_0 + (1/2) \omega''(\beta_0)(\beta - \beta_0)^2, \qquad (1)$$

where  $\omega_0$  is the cutoff frequency and  $\beta_0$  is the wave number corresponding thereto.

We write the hf current and the longitudinal electric field component in synchronization with the beam within the TWT in the following manner:

$$\widetilde{I} = \operatorname{Re}\left[J(x, t) \exp\left(i\omega t - i\beta_0 x\right)\right], \quad \widetilde{E} = \operatorname{Re}\left[\mathscr{E}(x, t) \exp\left(i\omega_0 t - i\beta_0 x\right)\right].$$
(2)

Dispersion equation (1) corresponds to the following free wave equation:

$$\left[\frac{\partial}{\partial t} + i \frac{\omega''(\beta_0)}{2} \frac{\partial^2}{\partial x^2}\right] \mathscr{E}(x, t) = 0,$$
(3)

As was shown in [2], the interaction of such a wave with the hf current is described by the equation

$$\left[\frac{\partial}{\partial t} + \frac{i\omega''(\beta_0)}{2} \frac{\partial^2}{\partial x^2}\right] \mathscr{E}(x, t) = -\frac{\omega_0 \beta_0 R(\beta_0)}{2} J(x, t), \qquad (4)$$

where  $R(\beta_0)$  is the coefficient of proportionality between the square of the field amplitude and the stored energy density.

In the case of a harmonic input signal the hf field and the current are proportional to  $e^{i\omega t}$ . We then take  $\mathscr{E}(x,t) = \mathscr{E}(x)e^{i(\omega-\omega_0)t}$  and  $J(z, t) = J(x)e^{i(\omega-\omega_0)t}$ , so that Eq. (4) takes on the form

Saratov State University. Translated from Izvestiya Vysshikh Uchebnykh Zavedenii, Radiofizika, Vol. 27, No. 8, pp. 1049-1055, August, 1984. Original article submitted April 6, 1983; revision submitted February 9, 1984.

$$\frac{d^2\mathscr{E}(x)}{dx^2} + \frac{2(\omega - \omega_0)}{\omega''(\beta_0)} \,\mathscr{E}(x) = \frac{i\omega_0\beta_0R(\beta_0)}{\omega''(\beta_0)} \,J(x). \tag{5}$$

This is the steady-state equation for excitation of the complex amplitude of the synchronous field  $\mathscr{E}(x)$  by a specified current  $J(x)e^{-i\beta \circ x}$ .

In the kinematic equation the interaction of the current waves with the field waves is described by Eq. (1)

$$\left[\frac{d}{dx}+i\left(\frac{\omega}{\upsilon_0}-\beta_0\right)\right]^2 J(x)=i\frac{\omega_0 J_0}{2u\upsilon_0}\mathscr{E}(x).$$
(6)

Further, we introduce dimensionless parameters

$$\begin{split} \underline{\xi} &= \epsilon \beta_0 x, \qquad \Omega = \frac{2 \left( \underline{\omega} - \underline{\omega}_0 \right)}{\epsilon^2 \beta_0^2 \left| \underline{\omega}''(\beta_0) \right|}, \qquad I = \frac{J}{I_0}, \qquad F = \frac{\mathscr{E}}{2\beta_0 u \epsilon^2}, \\ \epsilon^4 &= \frac{I_0 R\left(\beta_0\right)}{2u} \frac{\underline{\omega}_0}{\beta_0^2 \left| \underline{\omega}''(\beta_0) \right|}. \end{split}$$
(7)

Assuming that the interaction parameter  $\varepsilon$  is small ( $\varepsilon \ll 1$ ), we retain in Eqs. (5), (6) only the terms of first nondisappearing order in  $\varepsilon$ :

$$\mu\left(d^{2}F/d\xi^{2}\right) + \Omega F = iI; \tag{8}$$

$$[d|d\xi - iB]^2 I = iF, \tag{9}$$

where  $B = (\beta_0 v_0 - \omega_0)/\epsilon\beta_0 v_0$  is a parameter characterizing the desynchronization of the beam with a wave of critical frequency:  $\mu = \text{sign}\omega''(\beta_0)$ .

3. System (8), (9) is of fourth degree, so it must be supplemented by four boundary conditions. We will establish the form of these boundary conditions for the total signal suppression regime.

The first two conditions are obvious — at the input end of the device current waves are not excited in the beam:

$$I(0) = 0, \qquad I'(0) = 0. \tag{10}$$

The two remaining conditions refer to the field, so to clarify their form we will consider a system with no beam. The field distribution in such a system conforms to the equation

$$\mu \left( d^2 F / d\xi^2 \right) + \Omega F = 0. \tag{11}$$

since there are no sources at the output end of the device, and there is no external signal, second-order equation (11) must be supplemented by a homogeneous boundary conditions. The general form of this condition is as follows:

$$\alpha_1 F(l) + \alpha_2 F'(l) = 0, \tag{12}$$

where l is the dimensionless length of the device,  $\alpha_1$  and  $\alpha_2$  are coefficients which are determined solely by the energy output construction and may be frequency dependent.

Boundary condition (12) obviously retains its form for a system with a beam. Since the condition for total signal suppression is equality to zero of the field at the device output,

$$F(l) = 0, \tag{13}$$

then, according to Eq. (12) the derivative also vanishes:

$$F'(l) = 0.$$
 (14)

In the language of propagating waves conditions (13), (14) indicate equality of zero of the amplitudes of incident and reflected waves.\*

As will be shown below, the boundary problem formulated has nontrivial solutions only for certain values of the parameters  $l = l(\Omega)$  and  $B = B(\Omega)$ . These suppression conditions as well as the corresponding field and current distributions over system length will not depend on power matching conditions at the input and output, since no parameter characterizing these conditions appears in the formulation of the problem.

\*A more detailed discussion of boundary conditions is presented in the appendix.



4. Upon imposition of the two boundary conditions (10), Eq. (9) has two linearly independent solutions which we will denote by subscripts 1 and 2. It will be convenient to impose the following conditions on these solutions (and system (10)):

$$F_1(0) = 0, \quad F'_1(0) = 1, \quad F_2(0) = 1, \quad F'_2(0) = 0.$$
 (15)

We will seek a solution of boundary problem (9), (10), (13), (14) in the form

$$F = C_1 F_1 + C_2 F_2.$$

$$F' = CF'_1 + C_2 F'_2.$$
(16)

The conditions for total signal suppression, Eqs.(13), (14), are satisfied if and only if the determinant of system (16)

$$D(\xi) = F_2(\xi) F'_1(\xi) - F_1(\xi) F'_2(\xi)$$

vanishes at  $\xi = l$ . The solution of the boundary problem is then given by

$$F(\xi) = C \left[ F_2(l) F_1(\xi) - F_1(l) F_2(\xi) \right], \tag{17}$$

where C is an arbitrary constant.

The determinant D is a function of the parameters l,  $\Omega$  and B. For fixed l and  $\Omega$  the equation  $D(l, \Omega, B) = 0$  defines some closed line (hodograph) in the complex D-plane. This line begins (as  $B \rightarrow -\infty$ ) and ends (as  $B \rightarrow \infty$ ) at the point (0,1), since at  $|B| \gg 1$  the beam and wave do not interact, and the quantities  $F_1$ ,  $F'_1$  and  $F_2$ ,  $F'_2$  are equal to their initial values Eq. (15). For  $l \ll 1$  the entire hodograph is located in the vicinity of the point (0,1). For this reason it can be said that if at some  $l = l_0$  the hodograph encloses the origin, there must exist values of the parameters  $l = l_p < l_0$  and  $B = B_p$  which causes D to vanish. (We assume the parameter  $\Omega = \Omega_p$  fixed.)

To construct the hodograph it is necessary to find solutions 1 and 2. The first can be found by numerical solution of the corresponding Cauchy problem for Eq. (9). Since as can easily be seen the second solution is obtainable from the first by differentiation, the latter can be reconstructed with the expressions

$$F_2 = F'_1, \quad F'_2 = \mu (-\Omega F_1 + iI_1), \quad I_2 = I'_1, \quad I'_2 = 2iBI'_1 + B^2I_1 + iF_1,$$
 (18)

which follow from Eq. (9).

Figure 1 shows the configuration of the hodograph in the D-plane for  $\mu = 1$ . (Since the hodograph is symmetric about the real axis, only the portion corresponding to B < 0 is presented.\*) The hodograph has the form of a spiral which expands more rapidly the larger l. Selection of the parameter l for a given  $\Omega$  will cause the hodograph to pass through the origin, thus defining the suppression condition. With increase in l the first, second, etc. turns of the spiral will pass through the origin, which corresponds to suppression regimes of first, second, etc. order.

\*Equations (8), (9) transform into each other if the sign of B is changed and values are replaced by their complex conjugates.



The proposed method for solution of the boundary problem is definitive, although the technique is numerical.

5. Figures 2 and 3 show the results of a calculation of total suppression conditions. The parameters used were not the quantities l, B, and  $\Omega$ , but rather their combinations  $l^4$ , Bl,  $\Omega l^2$ . The latter are more convenient in that  $l^4$  is proportional to current, Bl, to voltage, measured with the value corresponding to exact synchronism of the beam with a wave of critical frequency as a reference (i.e., Bl = 0 corresponds to intersection of the dispersion characteristics of beam and retarding system exactly at the cutoff frequency), and  $\Omega l^2$  is proportional to the difference between the external signal frequency and the cutoff frequency ( $\Omega l^2 = 0$  corresponds exactly to the cutoff frequency). Thus, in fact Figs. 2 and 3 show the dependence of suppression current and voltage on frequency. Figure 2 presents the corresponding curves for the low-frequency limit, and Fig. 3, for the high.

At  $|BI| \gg 1$  (large detuning of voltage from the critical point) the suppression conditions found correspond to conventional Kompfner breakoff (µB < 0) or start conditions of an RWT without reflections (µB > 0), the dashed lines of Figs. 2 and 3. Upon approach to the cutoff frequency these regimes change somewhat. The suppression regime realized near the high-frequency edge of the passband at B = 0 (Fig. 3b) is of special character. It has no analog in single wave theory and is caused solely by reactive interaction of beam and field (it is simple to show that at B = 0 the phase shift between current and field is exactly  $\pi/2$ ), and corresponds to a zero net energy flux in the input section of the device.

## APPENDIX

As is well known, the total electromagnetic field in a periodic waveguide can be represented in the form (3):

$$E(x) = \sum_{s} C_{s}(x) E_{s}^{0}(x) e^{-l\beta_{s}x} + \sum_{s} C_{-s}(x) E_{-s}^{0}(x) e^{-\beta_{-s}x}, \qquad (A1)$$

where  $E_{\pm s}^{o}$  are the waveguide eigenfunctions, and  $\beta_{\pm s}$  are the propagation constants for forward and reverse waves. We will limit ourselves to calculation of forward and returning waves of one type:

$$E(\mathbf{x}) = C_s(\mathbf{x}) E_s^0(\mathbf{x}) e^{-i\beta_s \mathbf{x}} + C_{-s}(\mathbf{x}) E_{-s}^0(\mathbf{x}) e^{-i\beta_{-s} \mathbf{x}}.$$
 (A2)

The eigenfields  $E_{\pm S}^{o}(x)$  are periodic functions of coordinate x, which permits their expansion in a Fourier series, i.e., transformation to representation of the field in the form of a set of spatial harmonics:

$$E(\mathbf{x}) = C_s(\mathbf{x}) \left[ \sum_{n} E_{sn} e^{-i\beta_{sn}\mathbf{x}} \right] + C_{-s}(\mathbf{x}) \left[ \sum_{n} E_{-sn} e^{-i\beta_{-sn}\mathbf{x}} \right].$$
(A3)

Let a periodic waveguide be loaded by some output device, with the complex reflection coefficient from that device being  $\Gamma(\omega)$ . Then

$$C_{-s}(L)\left[\sum_{n} E_{sn}e^{-i\beta_{sn}L}\right] = \Gamma(\omega)C_{s}(L)\left[\sum_{n} E_{-sn}e^{-i\beta_{-sn}L}\right],$$
(A4)

where L is the length of the device.

For the case of signal suppression, at the output the incident wave must vanish:

$$C_{s}(L)\left[\sum_{n}E_{sn}e^{-i\beta_{sn}L}\right] = 0.$$
(A5)

Conditions (A4) and (A5) are compatible only if

$$C_{s}(L) = 0, \qquad C_{-s}(L) = 0.$$
 (A6)

This is then the condition for total signal suppression. We note that Eq. (A6) implies vanishing of not only the incident wave, but also the total electromagnetic field at the device output.

We now separate from the total field E(x) the portion in synchronism with the electron flow  $E_{c}(x)$ :

$$E = E_{\rm c}(x) + E_{\rm inc}(x), \tag{A7}$$

where

$$E_{c}(x) = C_{s}(x)E_{sm}e^{-i\beta_{sm}x} + C_{-s}(x)E_{-sp}e^{-i\beta_{-sp}x}.$$
(A8)

Here m and p are the numbers of adjacent harmonics synchronous with the current; for example, for a system with negative dispersion of the zeroth harmonic near the low-frequency limit m = 1, p = 0.

The synchronous field  $E_{s}(x)$  is related to the complex amplitude  $\mathscr{E}(x)$  of Eq. (5) by the expression

$$\mathscr{E}(x) = E_{c}(x) e^{i\beta_{0}x}, \tag{A9}$$

where  $\beta_0 = \beta_{sm}(\omega_0) = \beta_{sp}(\omega_0)$  is the wave number at the critical frequency. Inasmuch as  $\beta_{sm} = \beta_0 + \Delta\beta$ , and  $\beta_{-sp} = \beta_0 - \Delta\beta$ , then

$$\mathscr{E}(x) = C_s(x) E_{sn} e^{-i\Delta\beta x} + C_{-s}(x) E_{-s\mu} e^{i\delta\beta x}.$$
(A10)

Eq. (A10) divides the amplitude of the synchronous field into the amplitudes of two harmonics.

We will return to Eq. (8). The latter can be divided into two excitation equations for the dimensionless amplitudes  $F_+$  and  $F_-$ , the two synchronous harmonics:

$$dF_{+}/d\xi + i\sqrt{\Omega}F_{+} = -I/2\sqrt{\Omega},$$

$$dF_{-}/d\xi - i\sqrt{\Omega}F_{-} = I/2\sqrt{\Omega}.$$
(A11)

(We consider here the low-frequency edge of the passband,  $\mu = 1$ ).

The amplitudes  $F_+$  and  $F_-$  are related to the corresponding dimensioned quantities (see Eqs. (2), (7), and [2]).

$$F_{+} = C_{s}(x) \frac{E_{sm}e^{-i\Delta\beta x}}{2\beta_{0}u\varepsilon^{2}}, \qquad F_{-} = C_{-s}(x) \frac{E_{-sp}e^{i\Delta\beta x}}{2\beta_{0}u\varepsilon^{2}}.$$
(A12)

It is clear that condition (A6) requires vanishing of the dimensionless amplitudes  $F_+$  and  $F_-$  at the device output

$$F_{+}(l) = 0, \quad F_{-}(l) = 0$$
 (A13)

where l is the dimensionless device length. We now add Eq. (All), obtaining

$$dF/dz + i \sqrt{\Omega} (F_{+} - F_{-}) = 0, \tag{A14}$$

where  $F = F_+ + F_-$ . But then it follows from Eq. (A13) that  $(dF/d\xi)$  (l) = 0. Moreover,  $F(l) = F_+(l) + F_-(l) = 0$ . We have arrived at boundary conditions (13), (14).

It should be noted that the derivation of boundary conditions presented herein relies on separation of the field into forward and returning waves and therefore is not applicable to the case of exact equality  $\omega = \omega_0$ . However, it is valid for frequencies as close as desired to the critical frequency (both high and low critical frequencies). The formulation of boundary conditions (13), (14) itself has no singularities at the critical frequency and can thus be extended to the case  $\omega = \omega_0$ . The derivation of the boundary conditions presented in the main text is free of this shortcoming, but is somewhat formal.

## LITERATURE CITED

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FORMATION OF AN ION BEAM IN A MULTIAPERTURE ION SOURCE

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UDC 621.382.002

The basic rules of formation of a beam of  $Ar^+$  ions 100 mm in diameter using a multiaperture ion source with electron oscillation are analyzed. It is established that the ion source, with spherical grids, forms a converging ion beam with an increased current density along the source axis. It is shown that the maximum ion-current density in the beam and the ion energy depend on the values of the potentials on the grids of the ion-optical system, the ratio of which determines the character of the ion-current density distribution over the area of the backing being treated.

The method of plasma-chemical etching of surface microrelief is now used extensively in the technology of the fabrication of large integrated circuits (LIC). At the same time, the method of ion-beam etching (sputtering), based on the physical interaction of ions of an inert gas, such as Ar<sup>+</sup>, with the LIC surface or target being treated, is becoming ever more popular [1, 2]. The method is used in operations of cleaning an LIC surface (with an ion energy  $E_i \leq 100 \text{ eV}$ ), for etching microrelief ( $E_i \sim 0.5 - 1.0 \text{ keV}$ , and for ionic deposition of metallic and dielectric films [3], and it has a number of advantages. First, it makes it possible to etch metals, dielectrics, and semiconductors at high speed without changing the working gas; second, it is characterized by a high process purity, assured by the absence of contact between the treated backing and the plasma zone and by running the process at a relatively low pressure in the vacuum chamber ( $\sim 4 \cdot 10^{-2} - 2.4 \cdot 10^{-3}$  Pa) it also makes possible the precise control of the process through the use of fully determined and easily varied values of the ion-current density, the energy of the ions, and the angle of their incidence on the treated surface.

Special multiaperture ion sources (MIS) with electron oscillation, used earlier as ion engines, have been developed to perform etching of microstructures on the surfaces of largediameter semiconductor backings and deposit thin films of various materials on them. They can form a large-diameter, multibeam ion stream and make it possible to obtain a higher ioncurrent density than with one-aperture sources at equal voltages on the electrodes [4]. Ion

Minsk Radio-Engineering Institute. Translated from Izvestiya Vysshikh Uchebnykh Zavedenii, Radiofizika, Vol. 27, No. 8, pp. 1056-1064, August, 1984. Original article submitted June 9, 1983.