SPATIAL STRUCTURES IN DISSIPATIVE MEDIA AT THE THRESHOLD OF CHAOS

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Multistable states of dissipative lattice systems at the threshold of dynamic chaos are investigated. A classification is developed for possible spatial structures and the mechanisms of their coexistence and formation are examined.

1. Introduction. Investigation of lattice models of spatially extended systems, constructed from elements exhibiting complex dynamics and transition to chaos, has made it possible to shed light on many fundamental questions in nonlinear physics (hyrodynamic turbulence, chaos in radio physics and electronic systems, nonlinear optics, etc.) [1-4]. On the other hand, lattice systems consisting of elements with complicated dynamics can be created artificially, for example in radio electronics or optics, in order to realize devices with new functional possibilities. An important property of these systems is that near the threshold of chaos they exhibit an enormous diversity of stable states, differing by their spatial structure. The existence of such multistability makes systems of this class promising from the standpoint of applications for information storage and processing.

The simplest system, from the viewpoint of both dynamics and practical implementation, is a lattice system consisting of elements exhibiting transition to chaos through period-doubling bifurcations and with dissipative coupling between neighboring elements. In physical applications this could be, for example, a chain of nonlinear oscillatory circuits with external periodic excitation. Dynamic processes in this class of lattice systems can be described with the help of the universal model

$$u_{n+1,m} = 1 - \lambda[u^2_{n,m} + \varepsilon(u^2_{n,m-1} - 2u^2_{n,m} + u^2_{n,m+1})]. \tag{1}$$

Here $u_{n,m}$ is the value of the dynamical variable at the m-th spatial site of the lattice at the n-th moment of the discrete time, λ is a parameter that controls the transition to chaos, and ε is the coupling parameter between neighboring subsystems. Such a model was proposed independently in [1, 5]. The numerical calculations described in [1] demonstrated that domains, whose number and size depend on the initial conditions, can arise in this model. In [5, 6] it is shown, on the basis of a renormalization-group analysis, that the model (1) is universals and scaling laws were found for the spatial structures forming on the path to chaos. In this paper we discuss the relation between these results, we give a classification of the spatial structures, and we study further the spatial structures and the mechanisms of their formation and coexistence.

2. Classification of Spatial Structures. It has been found that the spatial structures in a dissipative medium can be classified on the basis of the types of structures existing when the spatial elements of the medium (lattice cells) are decoupled. Each cell is a system that exhibits period-doubling bifurcations as the parameter λ increases. At first we choose a value of λ such that a cycle of period 2 is realized. Then each cell can undergo motion in one of two phases: $u_1, u_2, u_1, \ldots$ or $u_2, u_1, u_2, \ldots$. In the general case the medium will consist of a collection of domains — regions in which the cells fluctuate in one phase. We shall call them first-order domains.

Now let the parameter λ take on a value such that when the coupling is switched off a cycle of period 4 is realized. The elements of the cycle u_1, u_2, u_3, and u_4 form two clusters, which we shall refer to as clusters of the first level:

$$\underbrace{u_3 \quad u_1} \qquad\qquad \underbrace{u_4 \quad u_2}$$

two clusters of the first level.

Elements closest in magnitude to one another are combined in one cluster. For regions of the medium where the instantaneous states of the cells refer to the same cluster, we shall retain the term first-order domains, while regions in which the states are identical we shall term second-order domains.

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We now consider a cycle of period 8. Its structure is as follows:

two clusters of the first level

$$\overbrace{\underbrace{u_3 \quad u_7} \quad \underbrace{u_5 \quad u_1}} \qquad \overbrace{\underbrace{u_4 \quad u_8} \quad \underbrace{u_6 \quad u_2}}$$

four clusters of the second level.

We now define first- and second-order domains as regions of the medium where the states of the cells fall within the same cluster of the first or second level, respectively, and we shall term third-order domains regions where the states are identical.

The principle of classification remains the same for further period doublings. For a cycle of period 2^p the elements u_1, u_2, u_3, ..., u_{2^p} are distributed over clusters of levels $i = 1, ..., p - 1$. There are 2^i-clusters of the i-th level. Each such cluster contains elements of a cycle with numbers differing by 2^{i-1}. Domains of order i are domains in which the states of the cells of the medium belong to the same cluster of the i-th level.

At the critical point $\lambda_c = 1.401155$ the attractor of an individual mapping is a Cantor set, and in an extended medium domains of arbitrary order can be realized.

Domain structures can exist when the dissipative coupling between the cells of the medium is switched on, but the minimum spatial size of the domains is restricted, and the restrictions are all the stronger the higher the order of the domains.

Figure 1 shows an example of a coordinate–amplitude diagram, obtained in a numerical experiment with the model system (1) with $\varepsilon = 1/3$ and random initial conditions and boundary conditions such that the ends are free

$$u_{n,0} = u_{n,-1}, \qquad u_{n,L} = u_{n,L+1}, \tag{2}$$

which ensures symmetric continuation of the solution to the left and right beyond the limits of the interval under study. In the diagram the spatial coordinate m is plotted along the horizontal axis and the values of the dynamic variable u, used at different moments in time, are plotted along the vertical axis. Here seven first-order domains can be seen. The longest of these domains contains two second-order domains, one of which, in turn, contains two third-order domains.

In the transcritical region of the parameter λ in the point system, as is well known, an inverse sequence of mergings of attractor bands is observed. Under these conditions, in the medium the possibility of the existence of domains of high order is eliminated first, and then all lower-order domains are eliminated.

3. Scaling of Spatial Structures. On the basis of the renormalization-group analysis developed in [3-6], it can be asserted that the spatial structures studied here can exhibit definite similarity or scaling properties. In particular, the structure of the domain wall separating two domains of order p should acquire for large p a universal form, and its width increases as $2^{p/2}$. This can already be seen roughly from Fig. 1 by comparing the configuration of the domain walls of different order present there. Special numerical calculations confirm this property with high accuracy. For this, initial conditions from which there arises a configuration having at the center of the section of the lattice under study a domain wall of order p = 1, 2, 3, ... are imposed. Choosing the value of the parameter λ according to the rule

$$\lambda_p = \lambda_c + (\lambda_{p-1} - \lambda_c)/\delta, \tag{3}$$

where $\delta = 4.6692$, we compare for different p the configurations that are established after a sufficiently large number of preliminary iterations. It turns out that when represented in terms of the scaling variables $m \cdot 2^{-p/2}$, $u \cdot a^p$, $a = -2.5029$, they agree well already for $p \geq 2$.

The scaling property also suggests an approximate relation, determining the limit on the size of the domain of order p: $l \geq 2^{p/2}\varepsilon^{1/2}$.

Numerical calculations show that if a domain structure is formed in the medium, then the dynamics within each domain is, to a high degree, independent of the dynamics within other domains. In addition, the character of the dynamics within the domains is determined by two parameters: λ and the domain size L. This assertion was made and checked with high accuracy in [3, 5] for single-domain states of a finite section of the medium. It turns out that the form of the map of dynamic regimes in the parameter plane (L, λ) is universal and does not depend on the specific choice of the model and the type of local nonuniformities at the ends

TABLE 1. Largest Lyapunov Exponent Λ in a Spatially Extended System (1)

p	λ_p	N_p	Λ	$\Lambda \cdot 2^p$
0	2	480	0,387 ± 0,020	0,387 ± 0,020
1	1,543689	960	0,202 ± 0,005	0,404 ± 0,010
2	1,430357	1920	0,0997 ± 0,0024	0,399 ± 0,010
3	1,407405	2560	0,0519 ± 0,0016	0,415 ± 0,013
4	1,4024922	5120	0,0256 ± 0,0009	0,410 ± 0,015

of the system; in particular, they can be domain walls. For domains of the highest orders the map of dynamical regimes is obtained by a similarity transformation with rescaling of the vertical scale by a factor of δ^{p-1} and the horizontal scale by a factor $2^{-(p-1)/2}$.

According to [3, 5], the transition to chaos within a domain as the parameter λ increases occurs through a sequence of bifurcations of doubling of the temporal period. In the process, the bifurcation values of the parameter λ increase as the domain size decreases.

Chaotic regimes of the dynamics of the medium also exhibit characteristic scaling properties. The upper diagrams in Fig. 2 exhibit spatiotemporal chaos for values of the parameter corresponding to merging of bands in the point system: $\lambda = 2$ and $\lambda = 1.5437$. The initial condition was an almost uniform state with small random disturbances. Each picture contains configurations for 6 and 12, respectively, successive moments in time after a sufficiently large number of preliminary iterations, in order that the regime be statistically settled. The dynamic regimes under study should be similar from the standpoint of their statistical characteristics: According to the scaling rule the characteristic spatial scale in the second case should increase by a factor of $2^{1/2}$ and the characteristic scale of the dynamic variable decreases by a factor a. This is confirmed visually by the diagrams studied.

One possible method for checking quantitatively the scaling of chaotic states is based on comparison of the Lyapunov exponents for values of the parameter λ related by the rule (3). For example, the largest Lyapunov exponent in a sufficiently long system should behave as $\Lambda \sim C2^p$. Table 1 shows the results of a calculation of this exponent for the model (1) with $\varepsilon = 1/3$ and the boundary conditions (2). The length of the system L = 60, the values of the parameter λ correspond to the points of merging of the attractor bands of an individual mapping, and the initial conditions are almost uniform states with a small noise perturbation. After a sufficiently large number of preliminary iterations the largest Lyapunov exponent was calculated seven times for each value of λ_p. Each time N_p simultaneous iterations of Eq. (3) and the corresponding variational equation were performed. The data obtained were analyzed using the standard method and the average values of Λ and the 95% confidence intervals, presented in the table, were determined. Comparing the quantities $\Lambda 2^p$ in the fourth column of the table shows that they agree within the limits of accuracy of the estimate.

The scaling properties of the spatial correlation function and the spatial spectrum of chaotic states, realized at the merging points of the bands λ_p, were confirmed in [7].

4. Formation of Spatial Structures with Instantaneous Switching of the Controlling Parameter. The states shown at the top of Fig. 2 do not contain differentials between different attractor bands, i.e., they are single-domain. Thanks to chaos, however, spatial structures with a definite characteristic spatial scale — the correlation radius r_{char} — and a definite characteristic amplitude — u_{char} — continuously arise and vanish in them. According to the results of the renormalization-group analysis [3] these quantities are estimated as

$$r_{char} \sim (\lambda-\lambda_c)^{-\log 2/2\log\delta}, \qquad u_{char} \sim (\lambda-\lambda_c)^{\log|a|/\log\delta}.$$

We now assume that the parameter λ is switched instantaneously from the transcritical region into the subcritical region. As a result, domain structure can arise in the medium. If, in the process, the switching is performed from a single-domain state corresponding to the 2^p-band attractor, then domains of order not less than p + 1 can arise. The characteristic spatial scale of these domains is determined by the correlation radius of the starting state, and it is all the larger the closer the state is to the critical point.

The bottom diagrams in Fig. 2 illustrate the process of formation of domain structures when the controlling parameter is switched instantaneously from 2 to 1 (a) and from 1.5437 to 1.3106 (b). In the examples studied there arise spatial structures containing, respectively, seven first-order domains (a) and four second-order domains (b) of different extent. Different configurations of domain structures arose in other realizations.

We assume that the above mechanism of formation of spatial structures is of interest, first as a method for preparing, for example, in an experiment, different spatial structures when the initial conditions are not controlled, second, as a unique method for instantaneously freezing the chaotic structure in order to study the spatial statistical properties, and, third, as a practical method for preparing random states (a unique type of random number generator).

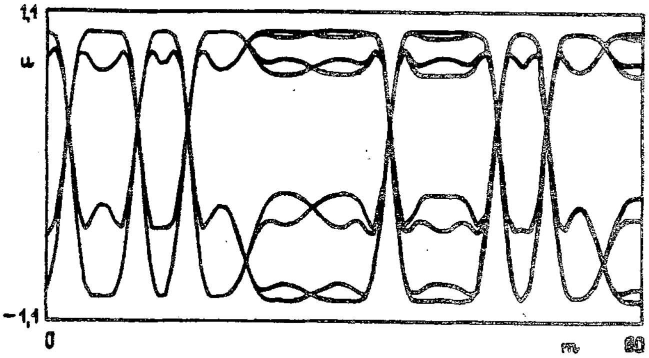

Fig. 1

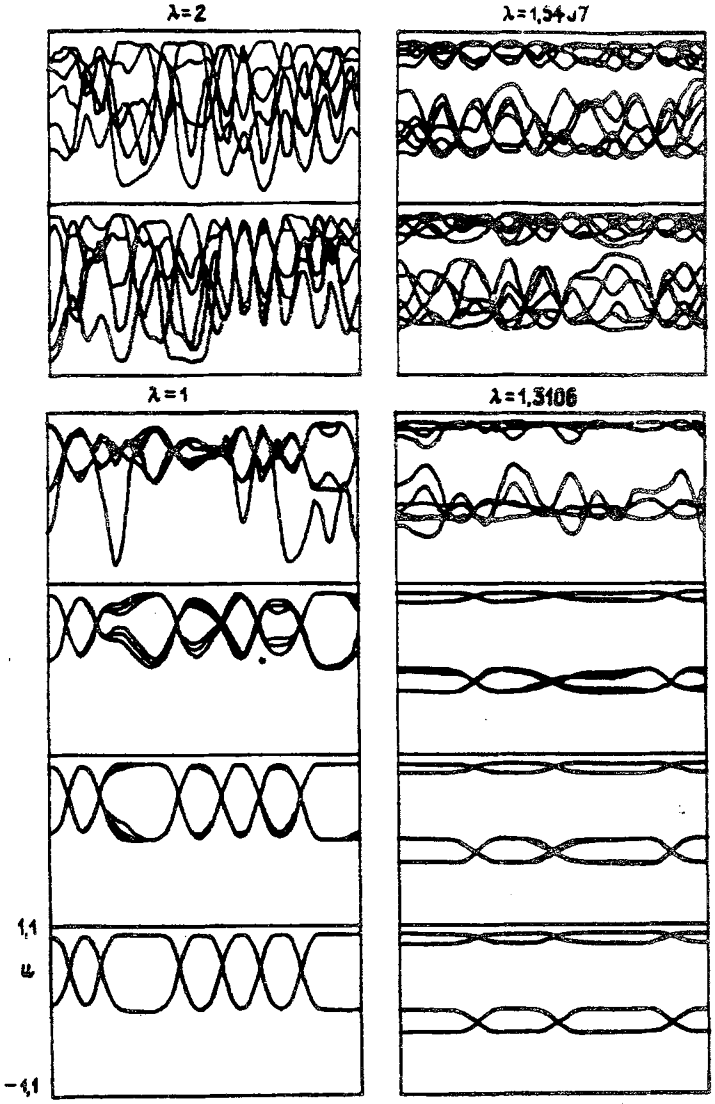

Fig. 2

The basic properties of spatial structures, realized at the threshold of chaos in a one-dimensional lattice model of a medium consisting of Feigenbaum systems, were investigated and demonstrated in a numerical experiment. A classification of these structures was given and the mechanisms of their appearance and coexistence, including scaling of the domain walls, were examined. Scaling of chaotic states of the medium in the transcritical region was illustrated qualitatively, the formation of domain structures accompanying instantaneous switching of the controlling parameter was studied, and the possibilities of applications of the latter effect were indicated.

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APPLICATION OF THE THEORY OF EXCURSIONS OF A RANDOM FIELD TO THE ANALYSIS OF RADIATION FROM A ROUGH SURFACE IN THE QUASISTATIC APPROXIMATION

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A phenomenological model is proposed and implemented for calculating the electrodynamic and radiative characteristics of a small-scale statistically rough surface. Elements of the theory of excursions of a random field are used, providing a simple means for determining the effective parameters of the transition layer from the given characteristics of the random surface. A numerical example is discussed, illustrating the main functional dependences of radiation from a Gaussian surface with a broad spectrum of roughness elements (corrugations). Restrictions are not imposed on the slopes of the corrugations.

The solution of practical problems of electromagnetic wave propagation across a rough (randomly corrugated) surface is generally based on the well-developed asymptotic methods of diffraction theory. For example, the methods of geometrical and physical optics, perturbation methods, and various numerical schemes are generally known [1-3]. However, major difficulties are encountered in the solution of diffraction problems involving densely clustered ($k_0 << K$) and steep ($Ka \geq 1$) corrugations ($K = 2\pi/\Lambda$, $k_0 = 2\pi/\lambda$, Λ and a are the horizontal and vertical dimensions of the corrugations, and λ is the radio wavelength). The impedance approximations are not always justified under these conditions [1, 4], but the more universal phenomenological (or quasistatic) approach can be used [5, 6]. It entails the introduction of a transition layer with effective parameters to model the rough interface between two media.

Simple quasistatic models of a rough surface in the rf range have been investigated previously [7-9]. They were tailored to one-dimensional (cylindrical) deterministic and random surfaces. The questions of macroscopic electrodynamics of two-phase media were used to calculate the effective parameters of the transition layer. In the present article we elaborate this theory for a randomly corrugated surface with a continuous two-dimensional spectrum of small-scale disturbances. A significant element of the model discussed below is the recourse to principles of the theory of excursions of a random field, which can be used to circumvent the difficulties encountered in the solution of the corresponding boundary-value problem on realizations of a random surface and to perform the requisite statistical operations for the ensemble of macroscopic parameters. As an example, we calculate thermal radiation from a water surface with anisotropic disturbances.

EFFECTIVE PARAMETERS

Let a stationary, spatially homogeneous perturbation field $z = \xi(x, y)$ be given with correlation function $B(r_2 - r_1) = B(\Delta x, \Delta y)$ or spectral density

$$S(K_x, K_y) = \int\int_{-\infty}^{\infty} B(\Delta x, \Delta y) \exp\left[-i(K_x \Delta x + K_y \Delta y)\right] d\Delta x d\Delta y.$$

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