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INFLUENCE OF A FRACTAL SIGNAL ON A FEIGENBAUM SYSTEM AND BIFURCATION IN RENORMALIZATION GROUP EQUATIONS

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UDC 517.9

A fractal signal model, having a phase portrait in the form of a two-scale Cantor set, is introduced to describe realistic signals generated by dynamic systems at the onset of chaos. It is shown that the effect of such a signal on a Feigenbaum system can lead to bifurcation of the renormalization group equation, consisting of coincidence and exchange of the nature and stability of its fixed points. From the point of view of real dynamics, this bifurcation corresponds to changing the type of critical behavior at the onset of chaos.

Substantial progress in nonlinear dynamics has been achieved recently in studying scenarios of transition to chaos [1]. Interesting examples of fractal attractive sets, generated in phase space of dynamic systems exactly at the onset of chaos, have been introduced. Fractals have a hierarchical organization, repeating in infinitely decreasing scales, and characterized by a fractional value of Hausdorff dimensionality [2, 3]. One of the simplest examples is the Feigenbaum attractor, realized at the critical point of transition to chaos in dynamic systems, demonstrating bifurcation cascades of period doubling. This attractor has the structure of a Cantor set with Hausdorff dimensionality $D_F = 0.538045$.

A powerful tool of theoretical investigation of similar situations is the renormalization group method, first developed in nonlinear dynamics by Feigenbaum [4], and further developed by many other investigators. This approach consists of constructing a special renormalization procedure, making it possible to determine, from some known evolution operator of a dynamic system at some time interval, the evolution operator in a time segment several times larger. By multiple application of this procedure one obtains a sequence of operators, describing the behavior of the system at longer and longer times. In phase space this corresponds to transition to treating the dynamics at smaller and smaller scales. In a critical situation the sequence of these operators has the limit of a universal operator, which also determines the structure of the fractal attractive set in phase space.

In view of the current achievements in understanding the behavior of critical dynamics of nonlinear systems, problems naturally arise concerning the mutual influence of motions of this kind. In this case both the dynamics of an acting system, and the dynamics of a system subject to action, are characterized by a hierarchical organization, covering a wide range of scales in phase space, no matter how small. Concurrent motions with different scale properties may or may not lead to new critical patterns in the dynamics of the second system. As we will show, a transition from one situation to another is possible in varying the parameters of an active signal, characterizing its scale properties.

Within the renormalization group method, this phenomenon is addressed as bifurcation of renormalized dynamics. Indeed, it can be assumed that the renormalized transformation provides some dynamics in operator space ("renormalized dynamics"), while for time ("renormalized time") there occurs a number of steps of renormalized transformations. A universal limiting operator is then associated with a fixed point of renormalized transformation, which, general-

Saratov Branch, Institute of Radio Science and Electronics, Academy of Sciences of the USSR. Translated from Izvestiya Vysshikh Uchebnykh Zavedenii, Radiofizika, Vol. 34, No. 6, pp. 661-670, June, 1991. Original article submitted February 8, 1990; revision submitted July 2, 1990.

 0033-8443/91/3406-0556 $12.50

ly speaking, can bifurcate in varying the problem parameters. Such a bifurcation must also occur in conventional dynamics, leading to a change in type of critical behavior, including rearrangement of structure of the fractal attractive set and changes in scaling constants.

1. Fractal Signal. A signal we would like to design so as to study, in what follows, its action on a nonlinear system must satisfy the following conditions. Firstly, it has a phase portrait in the form of a fractal structure and a hierarchy of scales, and, secondly, it contains parameters making it possible to control its scaling properties, and whose variation could generate bifurcation of renormalized dynamics.

We define a fractal signal as a sequence y_n, given in discrete time and constructed by means of the recursion relations

$$y_{2n} = b(1 + y_n), \qquad y_{2n+1} = -a(1 + y_n), \tag{1}$$

where $a < 1$ and $b < 1$ are positive constants. The initial element is $y_0 = b/(1 - b)$.

It can be shown that such a signal can be produced by assigning an appropriate rule of time dependence of elements of a two-scale Cantor set [2]. This set is obtained from the initial segment $[-a/(1 - b), b/(1 - b)]$ by means of the following recursion scheme. During the first step of the construction the segment is divided into three parts of ratio a, 1-a-b, b, and the mid interval is excluded. The same procedure is applied to each of the remaining segments, etc. The fractional dimensionality of the limiting set D is determined by the equation [2]:

$$a^D + b^D = 1. \tag{2}$$

We note that the given geometric treatment and Eq. (2) is valid if $a + b < 1$, since otherwise the intervals, of which the set is constructed, overlap. The original definition (1) remains sensible even when this condition is violated, but in that case the dimension is usually defined to be equal to unity.

For $a = 1/2.5029$ and $b = a^2$ the set constructed approximates the Feigenbaum attractor at the onset of chaos generation through period doubling, while the signal elements are numbered in the correct order of time sequencing during their construction by means of Eqs. (1). The Hausdorff dimensionality, calculated by Eq. (2), is in this case 0.5245, quite close to the exact D_F value.

The model (1) can be used similarly to describe signals, generated by mappings of more general form $y_{n+1} = 1 - \lambda|y_n|^z$ with arbitrary powers z for critical values of the parameter λ. For this one must put $a = 1/b$, $b = 1/\alpha^2$, where α is a scaling factor for given z [5]. For $a = b = 1/3$ the elements y_n span the points of the classical Cantor set, located in the segment $[-\frac{1}{2},\frac{1}{2}]$. Such a signal is naturally called a Cantor signal. For $a = 1$, $b = 0$ the elements y_n acquire only the values of 0 and -1, while their time-sequenced order corresponds to the sequence R and L in the symbolic trajectory description at the onset of chaos generation through period doubling [6].

Thus, many interesting fractal signals, generated naturally in dynamics of nonlinear systems or constructed artificially, are combined in one wide class, since they are obtained at least approximately by means of the common sceme (1) for different values of the parameters a and b. Though the suggested construction may be somewhat artificial, it is a useful tool for deeper understanding of the nature of critical effects in dynamic systems, and, in particular, it creates a basis for treating bifurcation of renormalized dynamics.

2. Dynamics of a Feigenbaum System under the Action of a Fractal Signal. Approximate Renormalization Group Analysis. Let the fractal signal (1) act now on a system demonstrating transition to chaos through period doubling. The situation is described by the model equation

$$x_{n+1} = 1 - \lambda x_n^2 + cy_n, \tag{3}$$

where x is a dynamic variable, λ is a control parameter of bifurcations of the original system, and c is a parameter of external action intensity. We note that multiple stability is, obviously, possible in system (3): for sufficiently small c values the system is capable of performing motion with different phase shift with respect to the fractal signal. We further restrict the discussion to situations for which at the moment of time $n = 0$ (see definition

(1)) the system is triggered from the vicinity of the origin of coordinates. Numerical calculations show that this attractor possesses the widest attractive basin.

We start with approximate renormalization group analysis in the style of [8], making it possible to obtain an explicit procedure of renormalized transformations and analyze the qualitative features of renormalized dynamics.

We construct the system evolution operator in two steps of discrete time. Considering only small values of the dynamic variable and of the external action intensity, we neglect terms containing x^4 and c^2:

$$x_{n+2} = 1 - \lambda(1 - \lambda x_n^2 + cy_n)^2 + cy_{n+1} \approx 1 - \lambda + 2\lambda^2 x_n^2 + (y_{n+1} - 2\lambda y_n)c. \quad (4)$$

We now use the definition of the fractal signal (1) and express y_n and y_{n+1} in terms of $y_{n/2}$, and also replace variables

$$x_n = [1 - \lambda - (a + 2\lambda b)c]X_{n/2} \quad (5)$$

and parameters

$$\lambda_1 = 2\lambda^2[\lambda - 1 + (a + 2\lambda b)c], \quad (6)$$

$$c_1 = c(a + 2\lambda b)/[\lambda - 1 + (a + 2\lambda b)c].$$

As a result we obtain the mapping

$$X_{n+1} = 1 - \lambda X_n^2 + c_1 y_n. \quad (7)$$

Thus, the evolution operator has been reduced following period doubling to the original form (3) with an accuracy up to variable replacement. This implies that the dynamics of two samples of the system, whose parameters are related by (6), are similar, differing in the time scale by a factor of two. The procedure discussed can now be applied to relationship (7), and the evolution operator can be obtained following four time steps, and so on. As a result of m-fold renormalization we reach a mapping of shape (3) with parameters λ_m and c_m, obtained recursively from the relations

$$\lambda_{m+1} = 2\lambda_m^2[\lambda_m - 1 + (a + 2\lambda_m b)c_m],$$
$$c_{m+1} = c_m(a + 2\lambda_m b)[\lambda_m - 1 + (a + 2\lambda_m b)c_m]^{-1}. \quad (8)$$

The pair of quantities (λ_m, c_m) can be considered as a representation of the evolution operator, since it is fully determined within the given approximation. Equation (8) is, therefore, also the renormalized dynamics equation.

Putting c = 0, one can find the fixed point of Eq. (8), corresponding to $\lambda_F = (\sqrt{3} + 1)/2$. This point is naturally called a Feigenbaum point, since it precisely determines the critical behavior of the system with inclusion of external action. There also exists a non-Feigenbaum point:

$$\lambda_{NF} = 1/(\sqrt{a^2 + 4b} + a), \qquad c_{NF} = 1 + 2\lambda_{NF} - 2\lambda_{NF}^2. \quad (9)$$

Linearizing the mapping (8) near fixed points, one can analyze the nature of their stability and determine the corresponding eigenvalues. For a Feigenbaum fixed point we obtain

$$\delta_F^{(1)} = 4 + \sqrt{3} \approx 5,73, \qquad \delta_F^{(2)} = 2\lambda_F(a + 2\lambda_F b), \quad (10)$$

and for a non-Feigenbaum point

$$\delta_{NF}^{(1,2)} = 1 - \lambda_{NF} + 2\lambda_{NF}^2(1 + bc_{NF}) \pm \sqrt{[1 - 2\lambda_{NF} + 2\lambda_{NF}^2(1 + bc_{NF})]^2 - 2\lambda_{NF}(3\lambda_{NF} - 2)}. \quad (11)$$

For $\alpha + 2\lambda_F b < 1/2\lambda$ the first point is a saddle point, and the second — an unstable nodal point. At the moment $\alpha + 2\lambda_F b = 1/2\lambda_F$ both points coalesce. For $\alpha + 2\lambda_F b > 1/2\lambda_F$ the Feigenbaum point becomes an unstable nodal point, and the non-Feigenbaum point — a saddle point. Thus, for $\alpha + 2\lambda_F b = 1/2\lambda_F$ there is bifurcation of coincidence and exchange of stability of fixed points, as well known in bifurcation theory [9]. The line combining both fixed points is a stable saddle manifold. The parameter values of the original mapping and of c, belonging to this line, correspond (within the approximation considered) to the critical situation of transition to chaos. Indeed, for given initial λ and c, multiple application of the renormalized transformation (8) at that line leads to displacement along it toward one of the fixed points. Till bifurcation this is a Feigenbaum fixed point, and following bifurcation – a non-Feigengaum point. The bifurcation line found within the approximation considered on the parameter plane of the fractal signal (a, b) is shown by dots in Fig. 1. The regions of Feigenbaum and non-Feigenbaum critical behavior are denoted by F and NF, respectively.

The analysis performed also makes itpossible to obtain approximate values of the critical indices. The constant corresponding to scaling by the control parameter is determined by the largest eigenvalue of the fixed saddle point:

$$\delta = \begin{cases} \delta_F^{(1)} = 5,73, & a + 2\lambda_F b < 1/2\lambda_F \\ \delta_{NF}^{(1)}(a, b), & a + 2\lambda_F b > 1/2\lambda_F \end{cases} . \tag{12}$$

The scale factor α is found by relation (5): $\alpha = \lambda - 1 + (a + 2\lambda b)c$. Substituting hence the λ and c values for saddle points of given a and b, we have

$$\alpha = \begin{cases} \alpha_F = 1/2\lambda_F = 2,73, & a + 2\lambda_F b </2\lambda_F \\ \alpha_{NF} = 2/(a + \sqrt{a^2 + 4b}); & a + 2\lambda_F b > 1/2\lambda_F \end{cases} . \tag{13}$$

The dependence of the scaling constants δ and α on the fractal signal parameter have a characteristic nodal point at the bifurcation point, related to changing the type of critical dynamics. Prior to bifurcation (shown below on the boundary of Fig. 1) it displays Feigenbaum behavior with fixed scale constants, and following it – non-Feigenbaum behavior with scale constants depending on the parameters a and b.

3. Exact Renormalization Group Analysis. We turn now to rigorous renormalization group analysis of the problem, which verifies the qualitative conclusions of the approximate theory and makes it possible to refine the values of critical indices and the bifurcation conditions in the plane of fractal signal parameters (a, b).

We initially obtain the exact renormalization group equations. We denote the first of (3) by $f_0(x, y)$, and turn to the mapping describing the change in state following two time steps:

$$x_{n+2} = f_0(f_0(x_n, y_n), y_{n+1}).$$

Changing the scale of the dynamic variable x by a number times $(-\alpha)$, and expressing y_n and y_{n+1} in terms of $y_{n/2}$ according to (1), we obtain a new mapping $f_1(x, y)$. Repeating the procedure multiple times, we reach the renormalization group equation

$$f_{m+1}(x, y) = -\alpha f_m(f_m(-x/\alpha, b(1 + y)), -a(1 + y)). \tag{14}$$

The fixed points of this equation are of further value. Obtaining these solutions implies finding the scale constant α and functions f(x, y), satisfying the equation

$$f(x, y) = -\alpha f(f(-x/\alpha, b(1 + y)), -a(1 + y)). \tag{15}$$

Seeking a solution of (14) in the form $f_m(x, y) = f(x, y) + \delta^m h(x, y)$, where h is a small correction, within the linear approximation we obtain

$$\delta h(x, y) = -\alpha[f'(f(-x/\alpha, b(1 + y)), -a(1 + y))h(-x/\alpha, b(1 + y)) + \tag{16}$$

$$+ h(f(-x/\alpha, b(1 + y)), -a(1 + y))].$$

Hence can be found the eigenfunctions $h_s(x, y)$ and eigvenvalues δ_s, characterizing the dynamics near the fixed point.

In our calculations the function f(x, y) was represented in the form of expansion in Chebyshev polynomials:

$$f(x, y) = \sum_{m,n=0}^{M,N} u_{m,n} T_{2m}(x) T_n(y). \tag{17}$$

(Since the function must be even in the first argument, polynomials in x with even subscripts only appear in the expansion.) To find the fixed points of Eq. (14) we used the Newton method, according to which the new approximation for the function f(x, y),

$$f_{\text{new}}(x, y) = f(x, y) + h(x, y),$$

is obtained by solving the equation

$$f(x, y) + h(x, y) = -\alpha f(f(-x/\alpha,\ b(1 + y)),\ -a(1 + y)) + \hat{L}h(x, y), \tag{18}$$

where $\hat{L}$ is the linear operator appearing in (16). In using expansion (17) Eq. (18) is converted to a system of M × N linear algebraic equations, while the matrix elements of the operator L in the corresponding basis were calculated by using the orthogonality of Chebyshev polynomials on a grid formed in the (x, y) plane by zeros of the product $T_{2M+1}(x)T_{N+1}(y)$. The 2M and N values in the calculations performed reached 10. The calculation results for several specific a and b values are given in Table 1.

To find the bifurcation coincidence of fixed points we substitute in (15) f(x, y) = g(x) + yh(x), where g(x) is the Feigenbaum function [4], and yh(x) is a small correction. Equating first order terms, we obtain the following existence condition of a doubly degenerate fixed point:

$$h(x) = -\alpha a[(b/a)g'(g(-x/\alpha))h(-x/\alpha) - h(g(-x/\alpha))]. \tag{19}$$

This problem is solved numerically as follows: the ratio $\xi = b/a$ is fixed, and by an iterative method one finds the largest eigenvalue in absolute value γ of the linear operator in the square brackets on the right hand side of (19). We further put $a = -1/\alpha\nu$, $b = -\xi/a\nu$. The bifurcation points in the parameter plane a, b are located on the solid line of Fig. 1.

4. How is Bifurcation of the Solution of Renormalization Group Equations Manifested in the Observed Behavior of the System? We now discuss the rearranged critical dynamics of the original system (3), accompanying the bifurcation found of the renormalization group equations, and expressed in the modification of scaling properties of the system at the onset of chaos.

With this purpose we have first investigated the dependence of the Lyapunov characteristic power on the parameter λ for system (3), referring to prebifurcation, the "Feigenbaum" situation. The critical transition point to chaos is fixed by transition of the Lyapunov power through zero. Unlike the traditional patterns, illustrating scenarios of period doubling, in the subcritical region there are no sharply expressed bifurcations: they are washed out by the effect of the external signal, containing components with all possible, 2^n-fold frequencies. However, with contraction of the neighborood of the critical point considered, the plot shapes approach the traditional ones, characteristic of the unperturbed logistic mapping [1]. According to the expected Feigenbaum scaling, the patterns obtained at high resolution possess scale invariance with respect to scale variation for $\delta = 4.669$ in the control parameter and twice in the Lyapunov power. The presence of the external signal is important only in the sense that critical parameter value depends on the action intensity c. The attractor at the critical point is a classical Feigenbaum attractor with dimensionality D_F.

The second, "non-Feigenbaum" situation is illustrated in Fig. 2. A sequence of plots is shown, demonstrating with increasing resolution the dependence of the Lyapunov power on the parameter λ near the critical point for the case a = 0.2, b = 0.2, c = 0.5. From plot to plot the vertical axis scale is recalculated twice, and along the horizontal axis it is converted by a factor 2.93239, corresponding to the older eigenvalue of the linearized renormalization group transformation at the non-Feigenbaum fixed point for given a and b. At the second step already the pattern practically stops changing. This verifies the presence of the expected scaling, and implies that the system dynamics at the onset to chaos is related to the non-Feigenbaum fixed point. A new fractal structure is formed at the critical point of transition to chaos: the total action of the external signal at all hierarchy levels leads to transformation of the Feigenbaum attractor into a new attractive set with different scaling properties. The Hausdorff dimensionality of this attractor depends now on the external signal parameters a and b.

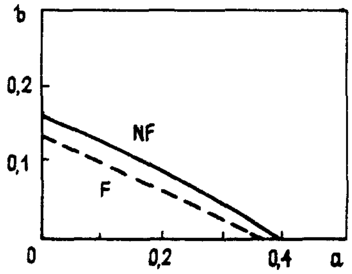

Fig. 1

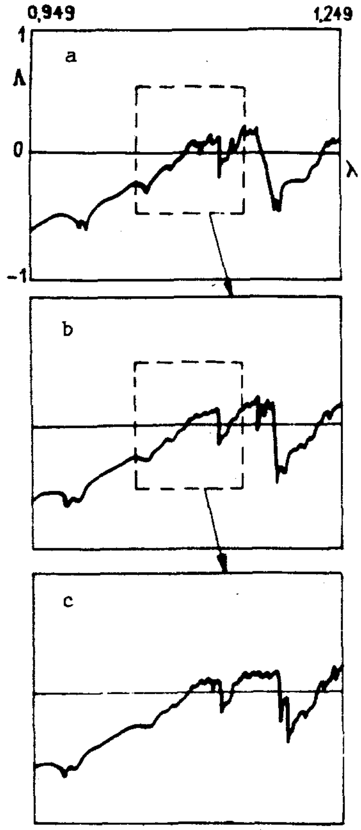

Fig. 2

TABLE 1. Scale Constants and Eigenvalues for Non-Feigenbaum Fixed Points of Eq. (14)

b	a	a_{NF}	$\delta^{(1)}_{NF}$	$\delta^{(2)}_{NF}$
0	0,28571	3,50000	9,451	1,232
	0,38462	2,60000	5,03506	1,03642
	0,43416	2,30329	3,99075	< 1
	0,5	2,00000	3,15823	
0,05	0,1	3,62884	10,08665	1,41811
	0,15	3,27345	8,08463	1,30943
	0,2	2,95726	6,52502	1,19965
0,1	0	3,16228	7,42181	1,36605
	0,1	2,74442	5,57055	1,14818
	0,3	2,04806	3,33476	< 1
	0,4	1,17701	2,75770	
	0,5	1,53259	2,39421	
0,15963	0,39954	1,54945	2,44700	< 1
0,2	0,1	2,03762	3,36808	< 1
	0,2	1,83305	2,93239	
	0,3	1,62654	2,57723	
0,3	0,1	1,68773	2,71621	< 1

5. Critical Dynamics near Bifurcation Points of the Renormalization Group Equation. Quasiscaling. As is well known, in traditional dynamics the time evolution of processes is slowed down upon approaching a bifurcation point. The same occurs for renormalization if the control parameters a and b are selected near the bifurcation curve of Fig. 1. The closer we are to a bifurcation point, the slower the renormalization at the stable manifold of fixed points. Within the approximate renormalization group analysis the equation of slow motion can be found by putting in the first of Eqs. (8) $\lambda_{m+1} = \lambda_m \approx \lambda_F$. We then obtain from it

$$\lambda_m = \lambda_F - \frac{a + 2\lambda_F b}{2 - 1/\lambda_F} c_m$$

and following substitution into the second of Eqs. (8) we have

$$c_{m+1} = 2\lambda_F(a + 2\lambda_F b)c_m - \frac{a + 4\lambda_F b}{2\lambda_F - 1} c_m^2. \tag{20}$$

One can now transform from the discrete renormalized time to the continuous

$$\frac{dc}{dm} = \varepsilon c - \frac{a + 4\lambda_F b}{2\lambda_F - 1} c^2, \tag{21}$$

where $\varepsilon = 2\lambda_F(a + 2\lambda_F b) - 1$ is a supercritical quantity.

In a real dynamic system (3) the approach to a bifurcation point of the renormalization group equation is expressed in such a manner, that to observe scaling related to one fixed point or another of the renormalization group equation it is necessary to descend to deeper levels of resolving the fractal structure. The required depth of this level can be estimated from the equation following from (21):

$$m \sim \text{const}/\varepsilon. \tag{22}$$

The situation generated in the immediate vicinity to a bifurcation point of the renormalization group equation can be characterized by the term quasiscaling: numerical calculations show that in transition from one level of resolution to another the plot of the Lyapunov power hardly changes, though a very slow evolution of these patterns does take place.

6. Explanation of Critical Behavior of Two Feigenbaum Systems with Unidirectional Coupling. A new type of critical behavior, called bicritical, was detected in [10] both numerically for a model system with two logistic mappings with unidirectional coupling, and experimentally for a system of unidirectionallly coupled nonlinear vibrational contours, excited by periodic external action. It is realized at a certain point in the plane of control parameters of the subsystems, and characterized by the fact that for arbitrarily small increase of parameters of the first or second subsystem chaos is, respectively, generated in the first or second subsystem. In the vicinity of this point there occurs a universal configuration of the regions in the parameter plane, characterized by two-parametric scaling: it transforms to itself upon scale changes along the coordinate axes by $\delta^{(1)} = 4.6692$ and $\delta^{(2)} = 2.39$ times. This corresponds to recalculating the scale of dynamic variables of the subsystems $\alpha^{(1)} = 2.5029$ and $\alpha^{(2)} = 1.51$ times.

These results can be explained within the larger context constructed in the present study. If the first system is located exactly at the onset of generation of chaos, it generates a fractal signal approximately described by model (1), to which corresponds some point in the (a, b) plane (Fig. 1). This point is located above the bifurcation curve, therefore the behavior of the second system upon transition to chaos is determined by the non-Feigenbaum fixed point with scaling constants equal, according to Table 1, to 1.549 and 2.447. It is seen that they are in good agreement with the constants $\alpha^{(2)}$ and $\delta^{(2)}$ found in [10].

7. Concluding Comments. We have, thus, shown that the following happens as a function of the parameters a and b, providing the scale properties of the external signal. For small a and b the transition to chaos in the system considered obeys Feigenbaum scaling with the classical values of scale constants $\delta = 4.6692$ and $\alpha = 2.5029$, independently of a and b. Following transition through some critical line in the a, b plane, the scaling properties of the dynamics at the onset to chaos start depending on a and b. This recalls the situation occurring in the theory of phase transitions, when the space dimensionality, considered as a continuous parameter, passes through the critical value $d = 4$. In this case Eq. (21) for slow renormalized dynamics in the vicinity of bifurcation points coincides, accurately within some coefficients, with the classsical Wilson-Fisher equation [7].

Having completed this analogy, one might think that the fractal dimensionality of the signal operating on the system must be the appropriate control parameter in our problem. This is, however, not the case: the bifurcation curve in the (a, b) plane does not coincide with lines of equal fractal dimensionality. This analogy has further methodological value. Indeed, it is known that including problems of the theory of phase transitions in a wider class, containing nonphysical situations, has proved useful for their understanding and approximate description. Exactly the same analysis of the effect of an artificially constructed signal (1) on the dynamics of a Feigenbaum system sheds light on the behavior of such systems under the action of realistic fractal signals.

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EFFECT OF RANDOM ATTENUATION OF RADIATION ON INTENSITY FLUCTUATIONS IN THE SATURATION REGION

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UDC 538.56

The propagation of electromagnetic waves in a turbulent medium with fluctuations of the complex dielectric permittivity ε is considered. Asymptotic expressions are obtained for the relative dispersion of intensity fluctuations σ_I^2 in the saturated region. It is shown that the presence of fluctuations in the imaginary part of ε leads to a substantial change in the nature of behavior of σ_I^2 for strong intensity fluctuations of radiation.

As is well known (see, for example, [1, 2]), in the classical theory of propagation of electromagnetic waves the medium is characterized by fluctuations in the real part only of the dielectric permittivity ε. Such a treatment, however, is not valid in all situations occurring in the atmosphere. In particular, in investigating the transmission of radiation through atmospheric paths, including aerosol layers with stable or varying microstructure, or investigating the propagation of radiation well-absorbed by gas components in a "pure" atmosphere, when the random variations in the imaginary ε_I and real ε_R components of ε are comparable, it is necessary to take into account the contribution of fluctuations in the imaginary part of ε to the statistics of the electromagnetic wave. Such problems were treated in [3, 4] for media with smooth correlation inhomogeneities in ε_R, ε_I within the approximate method of smooth perturbations in the cases of propagation of submillimeter radio waves in the atmosphere [3] and of laser beams in the channel of an illuminated cloudy medium [4], as well as in [5, 6] for the cases of optical radiation transfer in a turbulent atmosphere containing discrete inhomogeneities, with calculations of radiation intensity fluctuations in them carried out for the model of ideally absorbing particles. One must, however, keep in mind that under certain conditions on the statistical characteristics of radiation in a turbulent atmosphere — even

Scientific-Industrial Union "Taifun." Translated from Izvestiya Vysshikh Uchebnykh Zavedenii, Radiofizika, Vol. 34, No. 6, pp. 671-680, June, 1991. Original article submitted March 6, 1990.

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