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# Chaos-Controlling Technique for Suppressing Self-Modulation in Backward-Wave Tubes

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Received January 29, 2003

**Abstract**—A method for suppressing self-modulation in backward-wave tubes is proposed. Additional delay is introduced into the feedback circuit, owing to which the output signal amplitude affects the electron beam current that enters into the interaction space. Numerical simulations demonstrate that the operating current in the single-frequency oscillation mode may be increased roughly twofold. © 2003 MAIK “Nauka/Interperiodica”.

The treatment of microwave electron devices with long-term interaction as nonlinear distributed dynamic systems relies on the nonstationary nonlinear theory. Such an approach has been developed for backward-wave tubes (BWTs) of various modifications [1–3], traveling-wave tubes [4], and gyrotrons [5]. In particular, nontrivial bifurcations (loss of stability in single-frequency oscillations that is accompanied by self-modulation and dynamic chaos) have been discovered in conventional O-type BWTs [1, 6–12].

Nonstationary processes and complex dynamic modes in BWTs may be of practical interest. Specifically, a BWT operating in the chaotic dynamic mode may be used as a noise generator whose spectrum concentrates within a certain frequency range, the center frequency being tuned by the accelerating voltage [7–10]. In relativistic BWTs, a feature of transient oscillations (an initial spike of the field amplitude) can be used to increase the pulse generation efficiency [11, 12]. However, in many cases, self-modulation is a parasitic effect that makes difficult single-frequency generation of high power and efficiency, which would otherwise be achieved by increasing the operating current. To eliminate self-modulation, it was recommended, in particular, that high-space-charge operating modes be used [8]. Another way discussed in [13] is to apply a BWT with coupled guiding structures. In this paper, considering a BWT as a dynamic system, we make use of the idea of state stabilization that is referred to as chaos control in nonlinear dynamics.

This concept was first put forward in 1990 by Ott *et al.* from the University of Maryland [14]. They demonstrated the possibility of periodic dynamics, instead of random oscillations, being realized in a nonlinear system by applying weak controllable actions to an adjustable parameter of the system. Later, other chaos-controlling techniques intended for system stabilization

and/or directing a phase trajectory into a desired region have been proposed. One simple and often efficient method is to use delayed feedback [15]. To date, many examples of efficient chaos control have been demonstrated: in nonlinear oscillators [16], lasers [17], systems with spin-wave instability [18], biology and medicine [19], and space navigation [20].

As is well known, in a BWT (Fig. 1a), an electron beam moves at a speed that is close to the phase velocity of the wave, which provides efficient interaction. The group velocity of the wave is opposite to the beam, which produces internal feedback and absolute instability and also gives rise to self-sustained oscillations when the beam current exceeds a certain starting value. With a further increase in the current, self-modulation appears. Its mechanism is illustrated by the space–time diagram shown in Fig. 1b. Let the amplitude of the HF field at the left end of the system, where the electron beam is injected, be relatively high at a certain time instant I. This leads the beam’s electrons to rebunch along the line (characteristic)  $x - v_0 t = \text{const}$ : the HF current varies along the length as shown in the bottom panel. As a result, at the instant II, the amplitude of the current at the right end appears to be small. Then, on the line along which the wave packet propagates with the group velocity by the law  $x + v_g t = \text{const}$ , the field amplitude will be lower. Therefore, at the instant  $t \cong L/v_0 + L/v_g$ , the signal amplitude at the left end is minimal (instant III). The weaker field bunches the beam more effectively (top panel), and the current attains a maximum value on the corresponding characteristic at the right end (instant IV). As a result, the field reaches a maximum (instant V) at the left end within the time  $T \cong 2(L/v_0 + L/v_g)$ . This gives an estimate of the self-modulation period. Numerical calculations refine the factor in this expression: it appears to be close to 1.5 instead of 2.

Apparently, self-modulation may be suppressed by varying the beam current at the entrance into the interaction space so that it will increase when the field amplitude is near the maximum and decrease when the field amplitude approaches the minimum. Figure 1a schematically shows that this can be implemented by introducing an additional control circuit that uses the chaos-controlling technique with delay [15]. The HF signal picked up from the BWT output is detected and filtered. As a result, the signal envelope is extracted in the BWT operating mode for which self-modulation exists or may exist. Further, the signal is separated and fed to the input of a differential amplifier with a delay of about half the self-modulation period in one of the branches. The output signal of the amplifier is applied to the control electrode (grid) of the electron gun as an additional bias voltage, thereby controlling the beam current at the entrance into the interaction space.

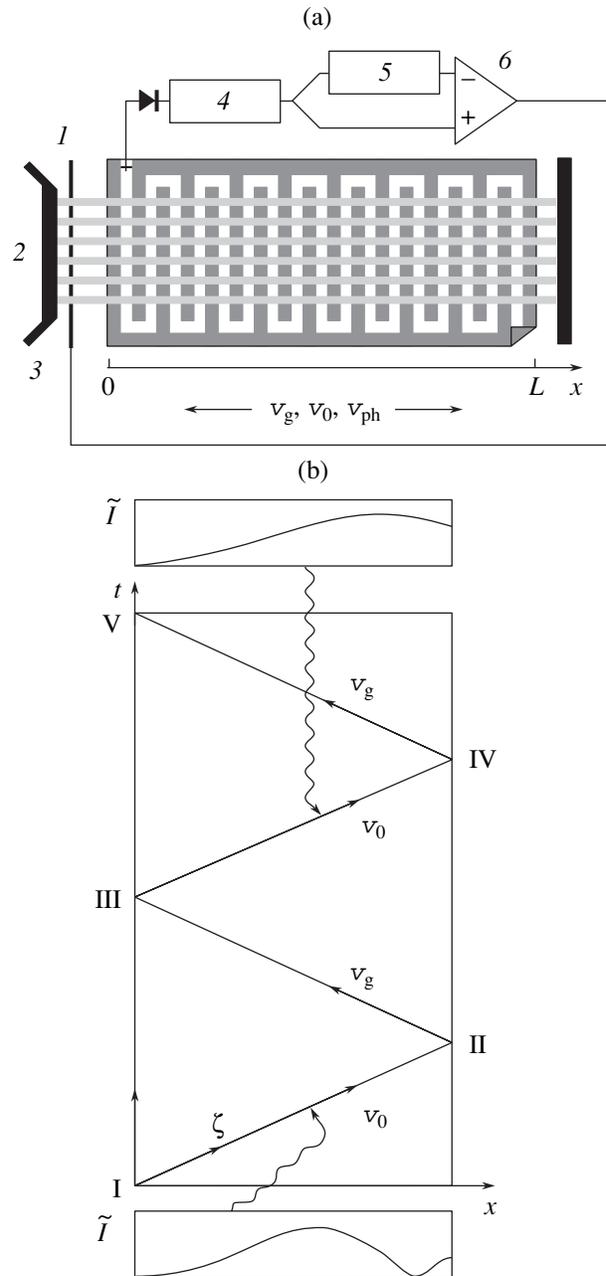
Let us confirm the possibility of suppressing self-modulation by a numerical experiment. To this end, we will invoke the equations of the nonstationary nonlinear theory of BWT [1, 8]. It is convenient to use the standard normalization of dimensionless variables and parameters, in terms of which the beam current is represented by the mean direct current  $I_0$ . The variation of the current due to the control circuit is allowed for by a factor on the right of the excitation equation. This factor is constant on the characteristic line that refers to the beam but varies in time, i.e., from characteristic to characteristic. The equations have the form

$$\partial^2 \theta / \partial \zeta^2 = -\text{Re} F \exp(i\theta), \quad \partial F / \partial \tau - \partial F / \partial \zeta = A(\tau) I, \quad (1)$$

$$I = -\frac{1}{\pi} \int_0^{2\pi} \exp(-i\theta) d\theta_0,$$

$$\theta|_{\zeta=0} = \theta_0, \quad \partial \theta / \partial \zeta|_{\zeta=0} = 0, \quad F|_{\zeta=l} = 0. \quad (2)$$

Here, the dimensionless independent variables  $\zeta = \beta_0 C x$  and  $\tau = \omega_0 C (1 + v_0/v_g)^{-1} (t - x/v_0)$  are defined so that the  $\zeta$  coordinate is directed along the characteristic (Fig. 1b);  $\beta_0$  and  $\omega_0$  are the wavenumber and circular frequency of the wave in the slow-wave structure when it is in synchronism with the electron beam;  $C = \sqrt[3]{I_0 K / 4U}$  is the Pierce parameter, which is expressed in terms of the beam current, coupling impedance  $K$  of the slow-wave structure, and accelerating voltage  $U$ ;  $F(\zeta, \tau) = \mathcal{E} / 2\beta_0 U C^2$  is the dimensionless complex amplitude of the HF field  $E(x, t) = \text{Re}[\mathcal{E}(x, t) \exp(i\omega_0 t - i\beta_0 x)]$ ; and  $\theta(\zeta, \tau, \theta_0)$  characterizes the electron's phase relative to the wave and refers to a particle that enters into the interaction space with a phase  $\theta_0$  and has a coordinate  $\zeta$  at the time instant  $\tau$ . In the absence of the control circuit, stationary oscillations occur when the dimensionless length is  $l > l_{st} \cong 1.974$  and self-modulation appears when  $l > l_{sm} \cong 2.9$  [1]. Note that the condition under which the field amplitude varies slowly in

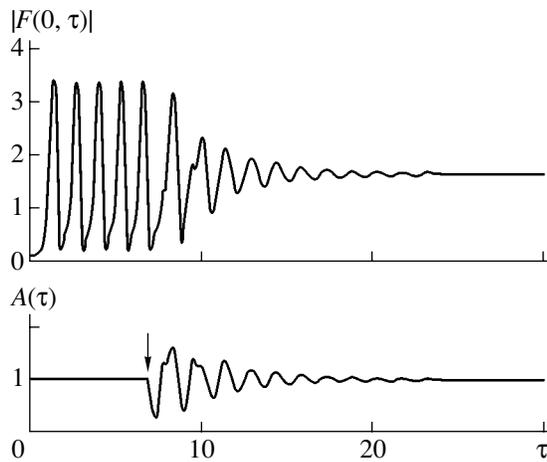


**Fig. 1.** (a) Diagram of a backward-wave tube (with the circuit that suppresses self-modulation by the chaos-controlling technique with delayed feedback) and (b) space-time diagram that illustrates the self-modulation mechanism: (1) microwave output, (2) cathode, (3) control electrode, (4) filter, (5) delay, and (6) amplifier.

time and space, which is fundamental for the applicability of the theory, is the smallness of the Pierce parameter.

Let us assume that, with the control circuit enabled, the electron beam current is given by the expression

$$J(t) = I_0 + g \Delta V, \quad (3)$$



**Fig. 2.** Dimensionless HF field at the BWT output and the normalized beam current versus time at  $l = 3.5$ . The switching time of the control circuit is indicated by the arrow. The parameters of the control circuit are  $c = 0.95$  and  $T = 0.8$ .

where

$$\Delta V = V(t) - V(t - \Delta t) \cong \beta_0^{-1} (|\mathcal{E}(0, t)| - |\mathcal{E}(0, t - \Delta t)|)$$

is the voltage at the output of the control circuit;  $V(t)$  is the amplitude of the HF potential at the BWT output;  $\Delta t$  is the delay; and  $g$  is a constant coefficient, which has the dimension of admittance.

Putting  $A = J/J_0$  and passing to the dimensionless variables, we can write

$$\begin{aligned} A(\tau) &= 1 + 2gUI_0^{-1}C^2(|F(0, \tau)| - |F(0, \tau - T)|) \\ &= 1 + cT^{-1}(|F(0, \tau)| - |F(0, \tau - T)|), \end{aligned} \quad (4)$$

where  $c = 2gUC^2/I_0 = \pi gKN$  and  $T = \omega_0 C(1 + v_0/v_g)^{-1}\Delta t$  are dimensionless constants, which characterize the control circuit.

Equation (1) in view of (2) and (4) was solved numerically by the finite-difference method [1, 8]. Figure 2 shows the output signal amplitude versus time at  $l \cong 3.5$ . The control circuit, initially disabled, was enabled at the time indicated by the arrow. It is clearly seen that the intense self-modulation decays and stationary single-frequency oscillations are established; i.e., the signal amplitude becomes constant. The additional terms in relationship (4) cancel out and  $A \equiv 1$ . Such an operating mode is also predicted by the stationary theory. However, the presence of the control circuit makes it stable. The empirical parameters of the control circuit,  $c = 0.95$  and  $T = 0.8$ , were selected such that the stationary oscillations occurred in a wide range of the dimensionless length  $l$ . For the above values of the parameters  $c$  and  $T$ , this situation takes place when  $l_{st} < l \leq 3.7$ . Thus, compared with an ordinary BWT, we managed to increase the self-modulation threshold in  $l$  by a factor of about 1.27 and by a factor of about 2 ( $1.27^3$ ) in the operating current with a minor change in

the dimensionless amplitude  $|F|$  of the output signal. Therefore, the maximum attainable single-frequency efficiency  $\eta = 2^{-5/3}I_0^{1/3}U^{-1/3}K^{1/3}|F|^2$  and power  $P = 2^{-5/3}I_0^{4/3}U^{2/3}K^{1/3}|F|^2$  grow, respectively, 1.3 and 2.5 times. Apparently, these parameters may be improved by applying more sophisticated control techniques.

To conclude, we note several points that should be taken into account when suppressing self-modulation in practice. First, the filter must reject the HF component of the signal and pass the fundamental self-modulation frequency (in the experiments reported in [6–9], these frequencies were 1 GHz and 50 MHz, respectively). Second, the delay must be about half the self-modulation period, i.e., about the time  $L/v_0 + L/v_g$  of signal travel through the feedback loop. Third, the transfer coefficient of the control circuit must be such that the input signal of amplitude on the order of the HF field amplitude causes the output current to vary by about the beam mean current. The magnitude of the HF voltage is estimated as  $C^2U$ , where  $C$  is a small parameter (in the experiments reported in [6–9],  $C = 0.005$ – $0.1$ ), whereas the voltage on the control electrode may apparently be lower than the accelerating voltage  $U$  by no more than one order of magnitude. Therefore, the presence of an amplifier in the control circuit seems to be necessary.

#### ACKNOWLEDGMENTS

This work was supported by the Russian Foundation for Basic Research, project no. 03-02-16192.

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*Translated by A. Khzmalyan*