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Forced Synchronization in a System with Unstable Cycle

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Abstract—The dynamics of a system with unstable limiting cycle under the action of a periodic sequence of delta pulses is considered. It is shown that stable quasiperiodic regimes and phase lock regimes (synchronization) can be observed within a narrow range of parameters of the external signal. *© 2003 MAIK "Nauka/Interperiodica".*

The phenomenon of synchronization, albeit known for a long time, still draws the attention of researchers. The classical case of synchronization consists in an external periodic (usually harmonic) signal acting upon an autooscillating system with a stable limiting cycle [1, 2]. In this case, frequency locking and quasiperiodic regimes are possible inside and outside the Arnold language, respectively, on the external signal frequency–amplitude plane. In phase space, these regimes are represented either by a stable torus (corresponding to a closed curve in the Poincaré cross section) or by the stable and saddle limiting cycles appearing on this torus upon crossing the language boundary.

Let us consider an alternative variant, whereby an unstable limiting cycle is realized in an autonomous system. At first glance, either unstable torus or unstable cycles can be expected in the phase space of this system in the presence of an external action. However, the nonlinear dynamics give many examples of phenomena related to the so-called problem of controlled chaos [3]. Within the framework of these notions, it is possible to study situations in which external actions can stabilize the initially unstable system. For example, an external signal acting upon a discrete system possessing an unstable cycle can lead to stabilization of this cycle, provided that the external signal is determined by elements of the unstable cycle. In this context, an interesting question is whether a pulsed action can lead to stabilization in a system with unstable limiting cycle and initiate stable synchronous and quasiperiodic regimes. We have established that such control is possible.

In order to demonstrate the possibility of induced synchronization, let us consider a system possessing an unstable limiting cycle, described by an equation of the Van der Pol–Duffing type and excited by a periodic sequence of δ-shaped pulses:

$$\ddot{x} + (\lambda - x^2)\dot{x} + x + \beta x^3 = B\sum \delta(t - nT). \qquad (1)$$

Here, x is a dynamic variable, λ is a control parameter (at $\lambda = 0$, the system features a reverse Andronov–Hopf bifurcation whereby an unstable focus at the origin becomes stable with separation of an unstable limiting cycle), T is the pulse repetition period, and B is the pulse amplitude.

In the autonomous case with positive λ, a system described by Eq. (1) possesses a stable immobile point and an unstable limiting cycle. The corresponding phase portrait is depicted in Fig. 1. Now let us switch on the external signal according to Eq. (1). Figure 2 shows a map of dynamic regimes on the T–B plane of parameters for this differential equation with the control parameter $\lambda = 1.2$ and the phase nonlinearity (nonisochronicity) parameter $\beta = 1$. In the map of Fig. 2, white color corresponds to the regime of period 1; light gray, to the regime of period 2, and so on; and black color indicates quasiperiodic regimes and chaos (the runaway of trajectories is also indicated by gray tint). The cycle period is determined in the Poincaré cross sections separated by time intervals equal to the period T of the external action.

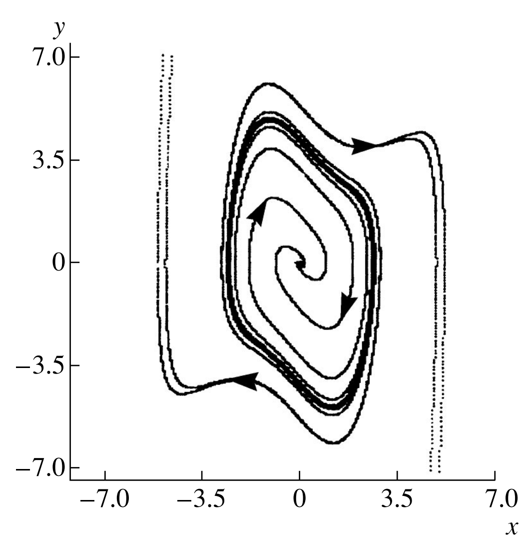

Fig. 1. The phase portrait of an autonomous system.

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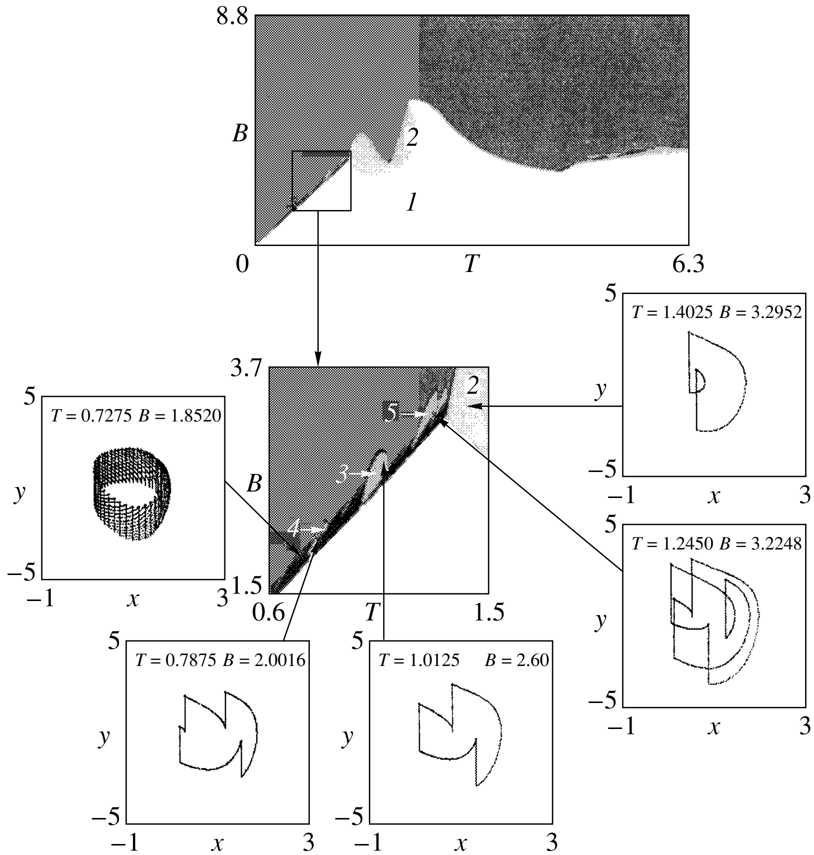

Fig. 2. The map of dynamic regimes of the differential equation (1) on the plane of parameters B–T for $\lambda = 1.2$ and $\beta = 1.0$. The bottom inset shows the indicated fragment on a greater scale and the corresponding portraits of attractors.

As can be seen from Fig. 2, there are two extended regions, one corresponding to stable motion with period 1 and the other to the runaway of trajectories. This is quite a natural physical result: the external action of a small amplitude drives the imaging point to a stable focus, while the pulses of a large amplitude drive the system beyond the boundaries of unstable cycle (whereby the imaging point goes to infinity). At the same time, there is a very narrow band of stable synchronization regimes separating the two regions. This synchronization band is shown in greater detail in the bottom part of Fig. 2, where the corresponding map fragments are depicted on a greater scale. As can be seen, the band contains quasiperiodic regimes and synchronization domains with periods 2, 3, 4, 5, etc. These regimes are also characterized by the portraits of attractors. The trajectory moves in the vicinity of the unstable limiting cycle of the autonomous system, while the external pulsed signal returns the trajectory to this vicinity. Thus, the system actually features a control situation initiating stable quasiperiodic regimes and the synchronization regimes in the vicinity of an unstable limiting cycle.

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