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THE EFFECT OF NOISE ON CRITICAL DYNAMICS OF DIFFERENT UNIVERSALITY CLASSES AT THE CHAOS THRESHOLD

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In many fields of mathematics and physics researchers use to classify entities according to their codimension, or degree of structural stability. In particular, this is a fundament of bifurcation theory and catastrophe theory. After Feigenbaum's discovery of the period-doubling universality and development of the renormalization-group (RG) method, it seems natural to use analogous approach to situations, which occur in multi-parameter families of nonlinear systems at the onset of chaos, and manifest scaling regularities (specific for each particular universality class). Now, a number of such situations has been discovered [1]. In concern to possible observation of these behaviors in physical systems, a question of principle significance is analysis of effect of inevitable noise, which destroys subtle details of the fractal-like self-similar structures in phase space and in parameter space associated with the transitions.

Theoretical approach to description of effect of noise based on the RG analysis was suggested in pioneering works of Crutchfield et al. and Shraiman et al. [2] in application to the period-doubling transition to chaos in dissipative systems. They obtained a universal factor γ=6.619036… responsible for the scaling properties of the transition in respect to the effect of noise. Namely, decrease of amplitude of noise with that factor ensures a possibility to observe one more level of the period doubling. In the following table we summarize relevant information for the case of the Feigenbaum critical behavior.

Critical point type: F (Feigenbaum) Scaling factors for state space and parameter space: α=–2.502907, δ=4.669201 **Noise scaling factor [2]: γ=6.619036**	Model map: $x \mapsto 1-\lambda x^2+\varepsilon\xi$ Critical point location: λ=1.401155189092

The above table is a pattern for other types of critical behavior, which are reviewed in this report. The noise is introduced in the equations by means of a random value ξ with $<\xi>=0$ and $<\xi^2>=1/12$.

For each type of critical behavior, we present respective universal constants responsible for scaling properties in respect to noise obtained from the RG analysis. The scaling properties are illustrated, e.g., with portraits of noisy critical attractors of model systems and gray-scale charts of Lyapunov exponents in the parameter planes in different scales (Fig.1). Also, we discuss feasible systems manifesting the critical behavior of the mentioned classes and possibility of experimental observation of the stated regularities.

Types of critical behavior intrinsic to one-dimensional maps

T (Tricritical) α=–1.690303, δ_1=7.284686, $\delta_2=\alpha^2$=2.857124 **Noise scaling factor: γ=8.243911**	Model map: $x \mapsto A-Bx+x^3+\varepsilon\xi$ Critical point location: A=0.242698757, B=1.594901356
S (six-power) α=–1.467742, δ_1=9.296247, $\delta_2=\alpha^4$=4.640870, $\delta_3=\alpha^2$=2.154268, **Noise scaling factor: γ=10.0378864**	Model map: $x \mapsto 1-Ax^2-Bx^4-Cx+\varepsilon\xi$ Critical point location: A=1.872448192, B=–1.625205285, C=1.094016102
E (eight-power) α=–1.358017, δ_1=10.948624, $\delta_2=\alpha^4$=3.401114, $\delta_3=\alpha^2$=1.844211, **Noise scaling factor: γ=11.593865**	Model map: $x \mapsto 1-Ax^2-Bx^4-Cx+\varepsilon\xi$ Critical point location: A=2.449366934, B=–1.260415730, C=0.700954625

Critical behavior of complex analytic maps at points of accumulation of M-tripling cascades

GSK3 (Golberg-Sinai-Khanin, period-tripling) α=–1.131475+3.260011i δ=–0.852664–18.1097279i **Noise scaling factor: γ=12.206641**	Model map: $z \mapsto A-z^2+\varepsilon\xi$ Critical point location: A=0.220536522+0.798246947i

Critical behavior intrinsic to period-doubling systems with unidirectional coupling

<u>**B (Bicritical)**</u> α=–2.502907, β=–1.505318, δ_1=4.669201, δ_2=2.392724 **Noise scaling factors: γ_1=6.619036, γ_2=2.713695**	Model map: $(x, y) \mapsto (1-\lambda x^2+\varepsilon_1\xi, 1-Ay^2-Bx^2+\varepsilon_2\xi)$ Critical point location at B=0.375: λ=1.401155189092, A=1.124981403

Types of critical behavior intrinsic to two-dimensional maps

<u>**H (Hamiltonian period-doubling)**</u> α=–4.018077, β=16.363897, δ_1=8.721097, δ_2=2 **Noise scaling factor [3]: γ=17.016534**	Model map: $(x, y) \mapsto (1-ax^2-by+\varepsilon\xi,\ x)$ Critical point location: b=1, a=4.136166804
<u>**FQ (Feigenbaum + Quasiperiodicity)**</u> α=–1.900072, β=–4.008158, δ_1=6.326319, δ_2=3.444710 **Noise scaling factor: γ=8.206143**	Model map: $(x, y) \mapsto (1-ax^2+dxy+\varepsilon\xi,\ 1-bxy)$ Critical point location at d=0.3: a=1.76719290, b=1.62967802
<u>**C ("Cycle", period-quadrupling)**</u> α=6.565350, β=22.120227, δ_1=92.431263, δ_2=4.192444 **Noise scaling factor: γ=51.06903**	Model map: $(x, y) \mapsto (a-x^2+by+\varepsilon\xi, -x^2+dy)$ Critical point location at b=–0.6663: a=0.24990280, d=0.452902880

Critical behavior of the golden-mean quasiperiodic orbits

<u>**GM (critical Golden-Mean quasiperiodicity)**</u> α=–1.288574, δ_1=–2.833611, δ_2=α^2=1.660424 **Noise scaling factor [4]: γ=2.306185**	Model (the circle map): $x \mapsto x + r - (K/2\pi)\sin 2\pi x + \varepsilon\xi$ (mod1) Critical point location: K=1, r=0.6066610634701
<u>**TDT (Torus Doubling Terminal)**</u> α^3=3.963766, δ_1=10.502983, δ_2=5.188118 **Noise scaling factor: γ=20.048638**	Model map: $(x, y) \mapsto (\lambda - x^2 + K\sin 2\pi y + \varepsilon\xi, y+w)$ Critical point at $w=(\sqrt{5}-1)/2$: λ=1.1580968567, K=0.3602484021

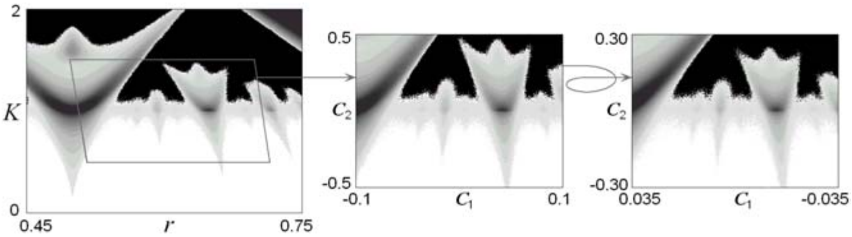

Fig. 1. Parameter plane scaling near the GM critical point in the circle map with noise. Gray tones designate negative values of the Lyapunov exponent Λ (the lighter the color, the less negative Λ is). Domains of zero Λ are white, and those of Λ>0 are black (chaos). The main diagram in coordinates (r, K) corresponds to the noise level ε=0.03. In the last two panels shown separately in special local coordinates (C_1, C_2) the horizontal and vertical scales are changed by $\delta_1=-2.833...$ and $\delta_2=1.660...$, and the noise level is decreased by $\gamma=2.306...$

Acknowledgements

The authors acknowledge support from grant of DFG and RFBR No 04-02-04011.

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