High-Dimensional Chaos in a Gyrotron

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Invited Paper

***Abstract*—In this paper, nonstationary and chaotic dynamics of a gyrotron with a nonfixed field structure have been studied. We discuss the mechanism of transition to self-modulation and chaotic oscillations and present the plane of parameters with different kinds of regimes in a gyrotron. For chaotic attractors, the set of Lyapunov exponents has been calculated. We have estimated the dimensions of chaotic attractors from the Kaplan–Yorke formula and found it to be anomalously high. The explanation of this fact is based on the presence of a large number of high-Q eigenmodes in the gyrotron resonator operating near a critical frequency of the electrodynamic system.**

***Index Terms*—Chaos, dimension, gyrotron, Lyapunov exponent, reflections, self-modulation.**

I. Introduction

RECENTLY, ideas and methods of nonlinear dynamics have been used extensively in application to distributed systems of the electron-wave nature, such as backward-wave oscillators (BWOs), gyrotrons, delayed feedback oscillators based on traveling-wave tubes (TWTs), and klystrons [1]–[3]. Now, it is established that for certain domains in parameter space, these systems can demonstrate single-mode or multimode chaotic oscillations. Analysis of Lyapunov exponents provides valuable complementary information concerning the nature of regimes observed in such systems.

One of the principal features of dynamic chaos is the property of exponential instability of phase trajectories [4]–[6]. This instability can be quantitatively characterized in terms of Lyapunov exponents. In a system with N-dimensional phase space, there is a spectrum of N such exponents. Their positive or negative values correspond to an exponential growth or decrease of small perturbations near a typical trajectory belonging to the given attractor. Chaotic attractors are characterized by the presence of one or more positive Lyapunov exponents.

In distributed nonlinear systems, such as electron-wave devices, the phase-space dimension is infinite, as is the total number of Lyapunov exponents of their attractors. However, in practice, only a limited subset of them is significant. Namely, we may restrict the number of Lyapunov exponents by taking into account only those that are enough to determine the so-called Lyapunov dimension of the attractor from the Kaplan–Yorke formula [4]–[6].

Only a few papers have been published containing application of the Lyapunov exponents for characterization of nonlinear dynamics in electron-wave devices. The first research of this kind was reported in [7], concerning chaotic oscillations in a BWO, and the largest Lyapunov exponent was estimated there. Later, a set of several Lyapunov exponents was calculated for a BWO model [8], and the dimensions of the attractors corresponding to chaotic regimes in this system were estimated. In [9], chaotic dynamics in a TWT model were examined, and estimates for several Lyapunov exponents were obtained.

Nonlinear dynamics of a gyrotron with a nonstationary longitudinal field structure were studied originally in [10]. By means of a numerical solution of the equations describing these dynamics, a possibility of self-modulation regimes in this system was established. Later, nonstationary processes in a gyrotron were simulated numerically in a broad range of parameters [11]. Recently, the self-modulation regimes have been observed experimentally in gyrotrons with delayed feedback (see, e.g., [2], [12], and [13]). Results of our recent numerical studies [14] show that transitions to chaos can proceed in accordance with various scenarios, either via a cascade of period-doubling bifurcations or via a destruction of quasiperiodic motions.

The aim of this paper is to present the main features of high-dimensional chaos in a gyrotron with a nonfixed longitudinal field structure. Section II contains a general statement of the problem. In Section III, we present the results of numerical simulation of nonstationary regimes and discuss the mechanism of transition to self-modulation and chaotic oscillations. Section IV contains numerical results for the spectrum of Lyapunov exponents in chaotic regimes of a gyrotron. The Lyapunov dimensions of the attractors for these regimes are estimated with the Kaplan–Yorke formula. We stress that the observed dimensions are much greater than those of chaotic attractors in other electron-wave systems studied so far. For the first time, this fact was reported in the letter [15]. Here, we present a more detailed study of this phenomenon by means of eigenmode analysis.

II. Basic Equations and Boundary Conditions

Our consideration of dynamics of a gyrotron with a nonfixed longitudinal field structure will be based on a numerical

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solution of the following system of equations written in terms of dimensionless variables and parameters [10]:

$$\frac{\partial^2 F}{\partial x^2} - i\frac{\partial F}{\partial t} = \frac{I_0}{2\pi}\int\limits_0^{2\pi} p(\varphi_0)\,d\varphi_0$$

$$\frac{dp}{dx}+ip\left(|p|^2+\Delta-1\right)=iF, \qquad p|_{x=0}=e^{i\varphi_0};\ \varphi_0\in[0\,,2\pi]. \tag{1}$$

Here, $F(x,t)$ is the complex amplitude of the high-frequency field, depending on the longitudinal coordinate x and time t, p is the transverse momentum of an electron, Δ is the mismatch between the cyclotron frequency and the critical frequency of the operating waveguide mode, and I_0 is the input electron beam current. The interaction equations are supplemented by an initial condition for the field: $F(x,0)=F_0(x)$.

The boundary condition at the entrance cross section of the device corresponds to the total reflection and is formulated easily as $F(0,t)=0$.

At the collector end, we consider that the interaction space turns into a horn with a small opening angle ψ used for energy output. As shown in [16] and [17], for this case, the boundary condition may be represented in the following form:

$$F(L,t) - i\kappa_0\int\limits_0^t G\left[\kappa_0^2(t-t')\right]\frac{\partial F(L,t')}{\partial x}dt' = 0$$

$$G(\tau)=\frac{1}{2\pi}\int\limits_{-\infty}^{+\infty} e^{-5\pi i/6}\frac{Ai'(e^{i\pi/3}\Omega)}{Ai(e^{i\pi/3}\Omega)}e^{i\Omega\tau}d\Omega \tag{2}$$

where $\kappa_0=(2\tan\psi/\nu_{\rm mn})^{1/3}2\beta_\parallel/\beta_\perp^2$ is determined by the opening angle of the horn ψ and the type of operating mode $\beta_{\parallel,\perp}=v_{\parallel,\perp}/c$, L is the dimensionless system length, $Ai(x)$ is the Airy function, and $\nu_{\rm mn}$ are the roots of the Bessel-function derivative: $J_n'(\nu_{\rm mn})=0$. The effective method for computation of the function $G(\tau)$ is given in [16].

Boundary condition (2) completes the formulation of the boundary problem, which governs the dynamics of the gyrotron model. Obviously, the presence of a nonlocal relation between the field and its derivative impedes the derivation of a numerical algorithm for solving the problem. However, estimates show that the dimensionless combination of parameters $\kappa_0 L$ for a typical geometry of a gyrotron cavity in realistic cases is rather large ($\gtrsim$10). Then, the nonlocal boundary condition (2) can be replaced by a simpler relation. Indeed, both the complex signal amplitude $F(x,t)$ and its spatial derivative are slow functions of the dimensionless time t, and the function $G(\tau)$ under the integral sign approaches zero rapidly as the argument increases. Hence, for large $\kappa_0 L$, the derivative can be taken out of the integral sign, and we obtain the following relation [14]:

$$\left.\frac{\partial F(x,t)}{\partial x}+i\bar{\kappa}_0F(x,t)\right|_{x=L}=0 \tag{3}$$

where $\bar{\kappa}_0=\kappa_0(\int_0^\infty G(\tau)d\tau)^{-1}=(0.6313-0.3645i)\kappa_0$. Note that boundary condition (3) coincides with the condition proposed in [18] to describe interactions in a coupled-cavity TWT. (A feature of our present case is that the parameter $\overline{\kappa}_0$ is complex.)

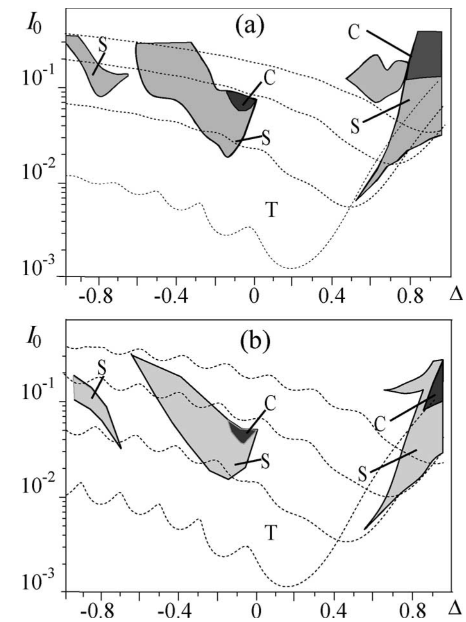

Fig. 1. Different regimes of oscillations on the plane (Δ, I_0): the white region above the dashed bottom border corresponds to stationary oscillations (T), and the gray areas designate self-modulation (S) and chaos (C). (a) Boundary condition (2). (b) Boundary condition (3).

The complete set of the formulated equations and boundary conditions contains four dimensionless parameters that determine the system behavior, namely 1) the frequency mismatch parameter Δ, 2) the dimensionless beam current I_0, 3) the system length L, and 4) the boundary condition parameter κ_0. In order to illustrate the results obtained, we assume that $\kappa_0 L = 15$ and $L = 14$. This dimensionless length is approximately equal to the length at which the highest possible efficiency is attained in the stationary theory of the gyrotron [19].

III. Complex Dynamics and Self-Modulation Mechanisms

We represent the results of examination of the nonstationary processes using a plane of two control parameters, namely 1) the mismatch parameter Δ and 2) the beam current I_0. Depending on their values, the system can exhibit stationary oscillations, self-modulation, or complex dynamics regimes. Fig. 1 shows the regions of various regimes on the parameter plane. The planes in Fig. 1(a) and (b) were obtained by using boundary conditions (2) and (3), respectively.

The regions of self-modulation and of chaotic oscillations are shown by shaded areas S and C. The area T corresponds to stationary generation that appears as a final result of some transient process. Dashed curves in Fig. 1 correspond to excitation of eigenmodes of the linearized gyrotron equations. Note that distinct modes have different thresholds. At one and the same value of the mismatch parameter Δ, there is a set of threshold currents I_0. The bottom envelope of the dashed curves determines the starting conditions for a single-mode generation.

The approximate relation (3) was derived under certain assumptions, and formally, at the chosen value of $\kappa_0 L = 15$, it can be used only for the frequencies $\Omega \leq 0.5$, which are close to cutoff (compare the frequency dependence of the reflection coefficient corresponding to the different boundary conditions shown in the inlay in Fig. 4). However, the numerical computations indicate that results obtained with the more accurate boundary condition (2) are qualitatively similar to those based on (3). The correspondence reveals itself in the presence of identical self-excitation mechanisms and routes to chaos and also in the approximate coincidence of the parameter values at which the self-modulation and chaotic regimes arise (see Fig. 1). The comparison allows us to conclude that the reduced boundary condition (3) is appropriate for description of the basic nonstationary phenomena in the gyrotron with satisfactory accuracy.

In Section IV, we will demonstrate, however, that a distinction in the behavior of the systems with boundary conditions (2) and (3) becomes notable from the point of view of some special features, such as dimensions of the chaotic attractors.

It is worth mentioning that the set of equations (1), (3) may be regarded as a model of some physical device in which the interaction between an electromagnetic field and an electron beam takes place in accordance with a gyrotron mechanism, but the construction of the energy output provides a good match only for frequencies in a narrow band near cutoff. In that case, the electrodynamic system will display strongly pronounced resonant properties for some frequencies distant from the critical one.

Transitions from stationary oscillations to self-modulation in distributed electron devices, including electronic microwave oscillators, were studied and discussed in a number of papers (e.g., review [20] and cited publications). As shown in [21]–[23], in TWT oscillators with delayed feedback and in distributed parametric oscillators, the transition to multifrequency oscillations takes place via either the amplitude mechanism or the phase one.

Results of the numerical solution of the nonstationary equations of the gyrotron model (1) indicate that the same two mechanisms of transition to self-modulation can take place, depending on the value of the mismatch parameter Δ [14]. At negative $\Delta \lesssim 0$, the amplitude mechanism occurs. In that case, the starting currents of the first several modes differ substantially, and the dynamics of the system are determined by the fundamental linear mode, which excites before others as the beam current I_0 is increased and Δ is held constant. In distributed electronic self-oscillators, the amplitude mechanism appears due to rebunching of electrons in a strong electromagnetic field, accompanied by reduction of the amplitude of the harmonic of the beam current at the collector end [21]–[23].

At large positive values of the mismatch parameter $\Delta \gtrsim 0.6$, the self-modulation develops due to the phase mechanism [21]–[23] intrinsic to resonant self-oscillators. It appears as a simultaneous generation of several eigenmodes of the system. At first, an initial distribution of the field in the system is transformed into intensive oscillations with an amplitude, frequency, and field distribution virtually time-constant and close to those associated with one of the linear eigenmodes. Then, as time evolves, these oscillations are destroyed by the nonlinear interaction with another mode, and the self-modulation regime of oscillation arises.

In correspondence with the two mechanisms of the birth of self-modulation, for further increase of the beam current I_0, one observes different scenarios for the onset of chaos. At $\Delta \lesssim 0$, the transition to chaos occurs via a cascade of period-doubling bifurcations. Since the dynamic regimes are based on the fundamental mode, we observe the period doubling of the self-modulation. At $\Delta \gtrsim 0.6$, the transition to chaos occurs via the destruction of quasiperiodic motion, involving the generation of several eigenmodes of the distributed cavity [14].

It has been established that, in addition to a chaotic regime characterized by a single positive Lyapunov exponent (weak chaos), the gyrotron can also demonstrate regimes with more than one positive Lyapunov exponent, which is referred to as developed chaos or hyperchaos [24]. In contrast to the case of weak chaos, the regime of hyperchaos has 1) essentially more homogeneous continuous spectrum with respect to the power distribution and 2) a phase portrait that is characterized by the absence of any pronounced structure.

IV. Lyapunov Dimensions of Chaotic Attractors

To compute the spectrum of Lyapunov exponents, we use a version of the Benettin algorithm [4]–[6] adapted to the distributed electron-wave system as described in [8]. A finite number of N exponents is obtained from a numerical solution of $N+1$ sets of (1) with boundary condition (2) or (3). The initial conditions for the field are defined as close complex functions $F_n(x) = F(x) + \varepsilon \widetilde{F}_n(x)$, where $\widetilde{F}_n(x)$ has a unit norm, and ε is small. After each time step $\Delta\tau$, the perturbations are orthogonalized and normalized with the Gram–Schmidt algorithm. Then, N values of the accumulating sums S_n are calculated at each step, which indicate a growth or decrease of a logarithm of a norm of the nth perturbation upon M steps of the procedure. That is

$$S_n = \sum_{i=1}^{M} \ln \left\| \tilde{F}_n(t = i \cdot \Delta\tau) \right\|. \tag{4}$$

From these sums, the Lyapunov exponents Λ_n are estimated as follows:

$$\Lambda_n = S_n / M\Delta\tau, \qquad n = 1, 2, \ldots N.$$

Once a sufficiently large number N of the Lyapunov exponents are calculated, an estimate of the dimension of the chaotic attractor may be obtained from the following Kaplan–Yorke formula [4]–[6]:

$$D = m + \frac{\sum\limits_{i=1}^{m} \Lambda_i}{|\Lambda_{m+1}|} \tag{5}$$

where m is determined by the condition that $\sigma_m = \sum_{i=1}^{m} \Lambda_i > 0$, whereas $\sigma_{m+1} = \sum_{i=1}^{m+1} \Lambda_i < 0$. The value D given by the formula (5) usually provides a good approximation

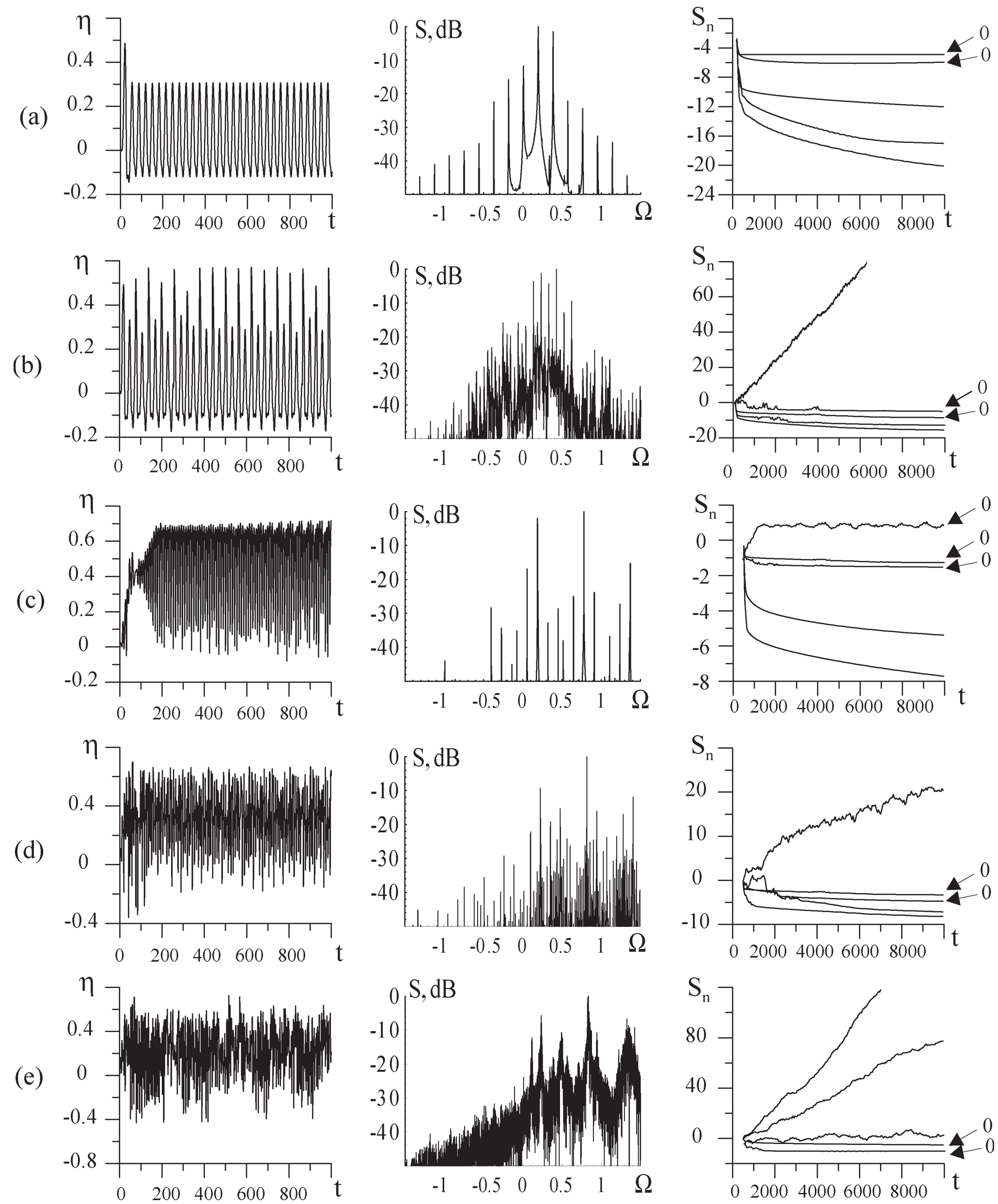

Fig. 2. Waveforms of electron efficiency $\eta(t)$, power spectra of the output signals, and plots of accumulating sums S_n versus time. (a) Self-modulation regime at $\Delta = 0$, $I_0 = 0.028$. (b) Chaotic oscillations at $\Delta = 0$, $I_0 = 0.038$. (c) Quasiperiodic self-modulation at $\Delta = 1$, $I_0 = 0.05$. (d) Chaotic oscillations at $\Delta = 1$, $I_0 = 0.12$. (e) Chaotic oscillations at $\Delta = 1$, $I_0 = 0.15$.

for the fractal dimension of a chaotic attractor. To be precise, D is called the Lyapunov dimension.

We calculated the Lyapunov exponents for different dynamical regimes in the gyrotron model described by (1) with boundary condition (2) or (3). Fig. 2 illustrates the results obtained via numerical solution of the equations with boundary condition (3). The first column presents the waveforms of electron efficiency $\eta(t)$ calculated from the transverse momentum of electrons at the collector end of the system $\eta = 1 - 1/(2\pi)\int_0^{2\pi}|p|^2 d\varphi_0$, the second column presents the power spectra of the output signal in the gyrotron, and the third column shows the accumulating sums S_n versus time. The following kinds of oscillations are presented: (a) the self-modulation regimes; (b) the regime of undeveloped (weak) chaotic oscillations, arisen after a sequence of period doubling bifurcations; (c) quasiperiodic self-modulation; and (d) and (e) the regimes of weak chaos and hyperchaos. Those $S_n(t)$ that increase with time correspond to positive Lyapunov exponents. It is worth noting that in our model, which was formulated in terms of complex amplitudes, any complex dynamical regime (self-modulation and chaos) has at least two zero exponents associated with perturbations of an infinitesimal phase shift and of a shift along the trajectory in the phase space of the system. In Fig. 2, these sums are indicated with "0" marks.

Fig. 3 shows the dependence of a sum of the first n Lyapunov exponents σ_n versus the number n. Curves 1 and 2 correspond to regimes of weak chaos at $\Delta = 0.0$, $I_0 = 0.0365$ [boundary condition (2)] and $\Delta = 0.0$, $I_0 = 0.038$ [boundary condition (3)], respectively. Curves 3 and 4 relate to regimes of developed chaos at $\Delta = 1.0$, $I_0 = 0.11$ [boundary condition (2)]

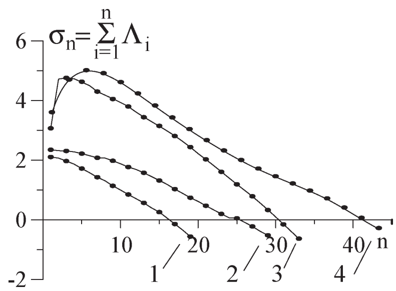

Fig. 3. Plots of the sums σ_n versus the number of Lyapunov exponents. Curves 1 and 2 correspond to "weak" chaos for $\Delta = 0.0$, $I_0 = 0.0365$ [boundary condition (2)] and $\Delta = 0.0$, $I_0 = 0.038$ [boundary condition (3)], respectively. Curves 3 and 4 correspond to developed chaos for $\Delta = 1.0$, $I_0 = 0.11$ [boundary condition (2)] and $\Delta = 1.0$, $I_0 = 0.15$ [boundary condition (3)], respectively.

and $\Delta = 1.0$, $I_0 = 0.15$ [boundary condition (3)], respectively. The point of intersection of each curve with the abscissa axis gives an estimate of the respective Lyapunov dimension by means of the Kaplan–Yorke formula. In a system with boundary condition (2), the attractor dimension in the regime of weak chaos is approximately $D \approx 16.4$, whereas that in the regime of hyperchaos is about 30.6. With boundary condition (3), the attractor dimensions are essentially larger: $D \approx 26.6$ for weak chaos and about 40.7 for the hyperchaos.

For comparison, we point out that estimates of attractor dimensions for a BWO presented in [8] in the regimes of weak and developed chaos were 3.5 and 6.4, respectively. For a TWT with delayed feedback, the estimate of the chaotic attractor dimension is $2 < D < 3$ [9].

Thus, the dimensions of chaotic attractors in our system are substantially higher than those obtained in other electron-wave systems. We suppose that this is linked with the fact that the gyrotron we consider operates near a boundary of the transmission band of the electrodynamic system.

To explain this assertion, let us turn to properties of the "cold" system (i.e., that in the absence of the electron beam). It corresponds to setting $I_0 = 0$ in (1). In this case, the equations have a trivial solution $F(x,t) \equiv 0$ that is obviously stable. Now, let us represent a field perturbation as a superposition of linear eigenmodes of the distributed resonator formed by the electrodynamic system, with the corresponding boundary conditions. Each mode will decay with time as $\exp(-\Omega_{\mathrm{im},n} t)$, where $\Omega_{\mathrm{im},n}$ is the imaginary part of the nth mode. Hence, the Lyapunov exponents calculated for the cold system are expressed by $\Lambda_n = -\Omega_{\mathrm{im},n}$.

Fig. 4 shows the disposition of the complex eigenmode frequencies for the cold system on the complex plane. The dots marked A and B relate to boundary conditions (2) and (3), respectively. As follows from analysis of the complex eigenfrequencies, the electrodynamic system has a large number of modes with relatively high Q (with a small positive imaginary part Ω_{im}). For boundary condition (3), the decay times of these modes are sufficiently large (and are of the same order) [15]. At this rate, the system will have a large number of negative Lyapunov exponents with small absolute values.

Depending on the type of the boundary conditions, these "cold" eigenmodes may contain either relatively high Q

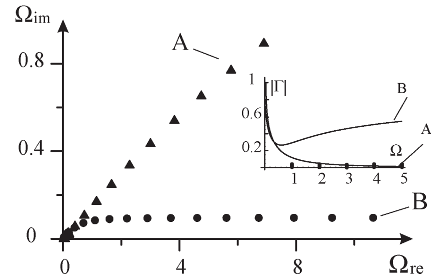

Fig. 4. Frequencies of the linear eigenmodes of a "cold" system on the complex plane $(\Omega_{\mathrm{re}}, \Omega_{\mathrm{im}})$. Curves A and B correspond to boundary conditions (2) and (3), respectively. The inset shows the absolute value of the reflection factor versus frequency for the aforementioned boundary conditions at the value $\kappa_0 L = 15$.

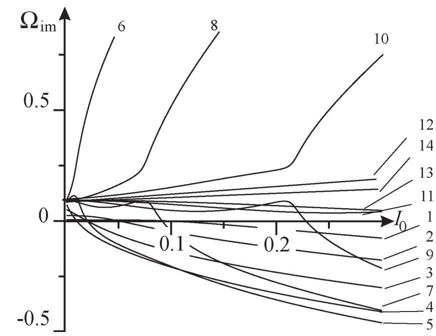

Fig. 5. Plots of imaginary parts of the linear eigenmodes of a system with the electron beam versus the beam current I_0 for boundary condition (3). The numbers at the curves indicate the numbers of the eigenmodes in ascending order of Ω_{re} at $I_0 = 0$.

[boundary condition (3)] or relatively low Q [boundary condition (2)], and the difference of imaginary parts of the frequencies of higher eigenmodes can be rather significant (see Fig. 4). For this reason, the attractor dimensions for the system with boundary condition (2) are notably lower than those in the case of boundary condition (3).

Let us examine the eigenmodes for a system with an electron beam using a linear approximation. The frequencies of the eigenmodes now depend on the beam current I_0. With a fixed mismatch parameter (e.g., $\Delta = 1.0$), one can monitor the imaginary parts Ω_{im} of linear eigenmodes varying with increasing beam current I_0. In Fig. 5, the dependences Ω_{im} versus I_0 are shown for boundary condition (3). As concluded, even taking into account the interaction with an electron beam, we have a large number of modes, which interact with the beam weakly over a wide range of beam current. For the mentioned modes, the imaginary parts of the eigenfrequencies are virtually constant or vary slightly with an increase of the current. In this case, the argumentation developed for the cold system remains valid.

V. Conclusion

We have carried out numerical simulations of nonstationary processes in a gyrotron with a nonfixed field structure. Boundary conditions that take reflections from the output horn into consideration have been formulated. On the parameter plane, the regions of different regimes have been established: 1) stationary oscillations; 2) self-modulation; and 3) chaotic oscillations. Numerical computations show that the transitions to self-modulation take place in accordance with amplitude or phase mechanisms, and the chaotic oscillations appear, respectively, via a sequence of period-doubling bifurcations or via the destruction of quasiperiodic motions. In addition to chaotic regimes characterized by a single positive Lyapunov exponent (weak chaos), the gyrotron also demonstrates regimes with several positive Lyapunov exponents. These regimes are referred to as developed chaos or hyperchaos.

Our research shows that the presence of a large number of high-Q eigenmodes in the situation of electron-wave interaction near the edge of the transmission band causes anomalously high Lyapunov dimensions of the chaotic attractors in a gyrotron. We can assume that the same phenomena will occur in other microwave self-oscillators operating near a cutoff frequency (e.g., relativistic orotrons and coupled-cavity TWTs).

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