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Picture of Pulsed Synchronization in the Dmitriev - Kislov Generator

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Features of synchronization picture in a system with a limit cycle embedded in a three-dimensional phase space are considered. By the example of Dmitriev - Kislov generator under the action of a periodic sequence of δ - function it is shown that synchronization significantly depends on the direction of pulse action. Features of synchronization tons appeared in this model are observed.

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1. Introduction

Synchronization is a fundamental nonlinear phenomenon. It is presently the subject of intensive interest in many areas of a science [1]-[8]. The classical case of synchronization consists in an external periodic (usually harmonic) signal acting upon an autooscillating system with a stable limit cycle [3], [6], [9] - [12]. In this case, frequency locking and quasiperiodic regimes are possible inside and outside the Arnold tongues, respectively, on the external signal frequency - amplitude plane. In phase space, these regimes are represented either by the stable torus or by the stable and saddle limit cycles appearing on this torus. However, there is a problems when external action has the form of short pulses. This external action can be represented as periodic sequence of $\delta-$ functions. Analysis of these problems is important both for the applications in many areas of science and for the theory of oscillations and nonlinear dynamics, where it is interesting to reveal possible specific features of the synchronization picture. In series of the works [13] - [19] this situations were studied within the framework of model system with a circle limit cycle under the action of the periodic sequence of δ - function. For this model approximated one-dimensional circle map for the phase is constructed. Authors was shown that synchronization picture both in a model system with a circle limit cycle and in a one-dimensional map had a number of features. This synchronization picture essentially different from the classical as in sine circle map [6], [20]. These features have a certain degree of universality, since they refer to the investigation of a system in the vicinity of the Andronov - Hopf bifurcation. For example, in the papers [21], [22] similar results was obtained for the Van der Pol and Van der Pol - Duffing oscillators under the action of the periodic sequence of δ - function. At the same time, in systems with two-dimensional phase space the external pulses can be directed along any direction in the plane, where the phase portrait of these systems represents a circle or the object close to circle. There are more complicated situation in the case then system with limit cycle embedded in tree-dimensional phase space [23].

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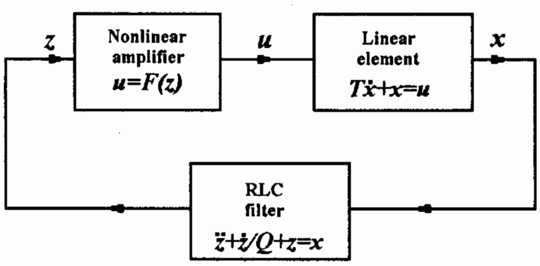

FIG. 1: Scheme of the Dmitriev - Kislov generator.

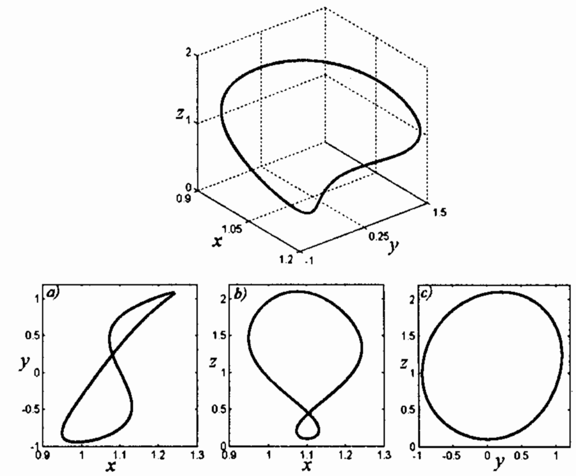

FIG. 2. Attractor and its projections for the Dmitriev - Kislov generator (1). Attractor and projections are plotted for $D = 6$, $M = 5.5$ and $Q = 10$.

Let there is a plane in which limit cycle is the circle (which is usually the case near the Andronov - Hopf bifurcation). Then, external pulse acts in any direction in this plane lead to described above results. Different situation takes place when the external pulse acts in the direction perpendicular to this plane.

2. Pulsed synchronization of the Dmitriev - Kislov generator

In the present work this problem is analyzed on an example of the reference model of nonlinear dynamics - Dmitriev - Kislov generator (Fig.1) [20], [24]:

$$\begin{aligned} x + D\dot{x} &= Mz\exp(-z^2), \\ \dot{y} &= x - z, \\ \dot{z} &= y - \frac{z}{Q}, \end{aligned} \qquad (1)$$

where x, y, z are the dynamical variables, D, M, Q are the system parameters. Dynamics of this system is studied adequately. Thus we selected the system's (1) parameters so that in phase space there is the period-1 stable limit cycle. Let us fix $D = 6$, $M = 5.5$ and $Q = 10$. The tree-dimensional attractor for system (1) and its projections are presented in Fig.2. One can see that projection of the attractor to the $(y,\, z)$ plane is circle (Fig.2c). Two other projections is different from the circle, essentially, projection to the $(x,\, y)$ plane. It is looks like the figure-of-eight (Fig.2a).

Let us add into Dmitriev - Kislov generator external pulses in the form $A\sum\delta(t - nT)$. Here A is the pulsed amplitude and T is the pulsed period. Note, that external pulses add each equations by turns. Thus, let us consider the following situations. First case, when pulses act along the x axis, that is external action add to the fist equation. Second case, when pulses act along the y axis, that is external action add to the second equation. Third case, when pulses act along the z axis, that is external action add to the third equation. Figure 3 shows the charts of dynamical regimes on the pulsed amplitude - period plane. These charts are plotted for all cases referred above. Chart of dynamical regimes is the plane of parameters, at which regions of periodic behavior with different periods are shown with different colors. In this and analogous charts to follow, the region painted white corresponds to the regime of period 1, bright-gray color corresponds to the regime of period 2 and above, black color refers to chaos and quasiperiodic regimes. As can be seen from Figs. 2 and 3, the case when pulses act along the y axis is most interesting. In this case synchronization picture differ both from the classical typical for the sine circle map and from the synchronization picture typical for system with a

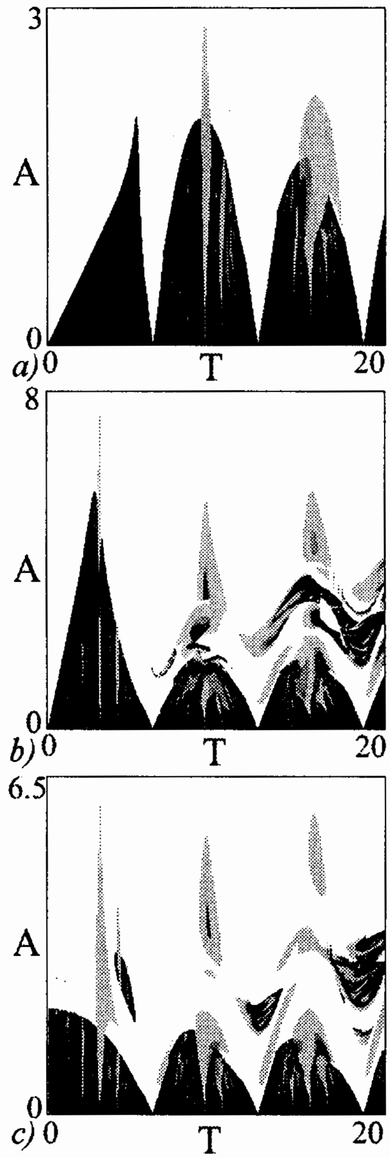

FIG. 3. Charts of dynamical regimes of the nonautonomous Dmitriev - Kislov generator on the A - T plane. Charts are plotted for the cases, when pulses act along the x axis (fig.a), the y axis (fig.b) and the z axis (fig.c). Systems parameters are fixed at $D = 6$, $M = 5.5$ and $Q = 10$.

circle limit cycle under the action of the periodic sequence of δ - function. We can explains this fact as follows. In the case then external action add to the second equation of the system (1) (to the dynamical variable y) pulses will be crossed both loops of the figure-of-eight (Fig.2a). In the other cases pulses are crossed either circle or one loop of the figure-of-eight. As a result, synchronization picture in the last two cases is the same like in the model with circle limit cycle under the action of the periodic sequence of δ - functions (Figs. 3a and 3b). Thus, let us discuss synchronization in nonautonomous Dmitriev - Kislov generator in the case then external pulses act along the y axis more detailed.

The fragments of the chart of dynamical regimes of nonautonomous attractors for the Dmitriev - Kislov generator are presented in the Figs. 4 and 5. In the charts one can see synchronization tongues corresponded to a periods multiple to the eigenvalue (the natural period of the system (1) $T \approx 6.22$). Region of a quasiperiodic regimes are surrounded these tongues. Higher-order synchronization tongues are observed in this region. Synchronization tongues corresponded to subharmonic resonance are located in the region of lesser periods of external action (comparison to the natural period). These synchronization tongues looks like strongly stretch up "drops". But note that, synchronization tongue with rotation number $\frac{1}{2}$ (period 2) differ from the others synchronization tongues in this region. This tongue is touched to $A = 0$ axis. First, it is converged in the middle part by forming "bridge". Then it is widen in the top part. Besides, in the top part of the chart this synchronization tongue comes to the period 1 region (Fig.4). Thus, on the one hand it is synchronization tongue with rotation number $\frac{1}{2}$. On the other hand it is the period 2 region, which is originated from the regime of period 1 being at large values of the pulsed amplitude as a result of period-doubling bifurcation. Nonautonomous attractors plotted for the Dmitriev - Kislov generator is verified this conclusion (Fig.4a-f). Thus, attractor plotted in the upper part of the period 2 synchronization region (Fig. 4b) looks like period 2 attractor originated from the period 1 attractor (Fig. 4a) as a result of period-doubling bifurcation. But, attractor plotted in the lower part of the period 2 synchronization region (Fig.

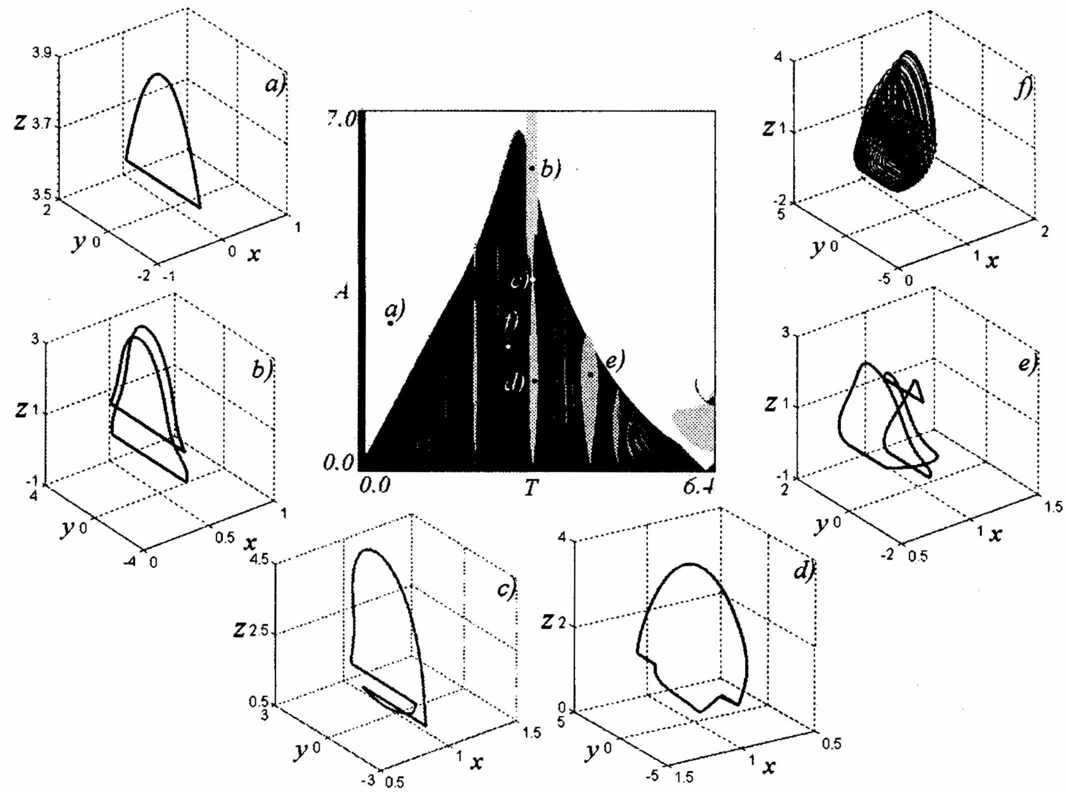

FIG. 4. Fragment of the chart of dynamical regimes for the nonautonomous Dmitriev - Kislov generator (1) from the Fig.3b. The portraits of nonautonomous attractors are shown in this figure too. Attractors are plotted for the next pulsed amplitude and period: a) $A = 3$ and $T = 0.8$; b) $A = 6$ and $T = 3.3$; c) $A = 3.7$ and $T = 3.13$; d) $A = 2$ and $T = 3.14$; e) $A = 1.5$ and $T = 4.1$; f) $A = 3$ and $T = 3$.

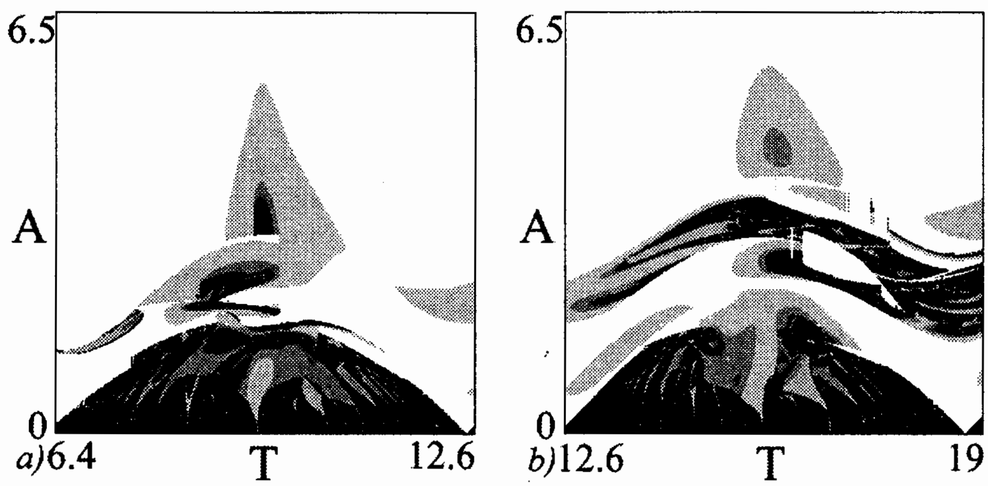

FIG. 5. Fragments of the chart of dynamical regimes for the nonautonomous Dmitriev - Kislov generator (1) from the Fig.3b.

4c) looks different. It is corresponded to regime of synchronization with rotation number $\frac{1}{2}$.

Transition between these to types of the nonautonomous attractors observed under decrease of pulsed amplitude. This transition consist in the next. One attractor loop (see Fig. 4b) is decreased throughout the height. While in the "bridge" it is situated in the same plane that action (Fig. 4c). After that loop is lower oneself under this plane (Fig. 4d). It is important note, that this process is accompanied by decrease of pulsed amplitude. These features are not observed for all others synchronization tongues in this region (Fig. 4).

Synchronization tongues of different periods and quasiperiodic regimes are observed on the right of main tongue (Fig. 5) However, these tongues are smaller by the amplitude and have more complicated structure. Inside these tongues there are regions of different period and chaos, i.e. transition to chaos through period-doubling cascade. Moreover, there are other regions of the complex dynamics in the top part of the charts presented in Fig. 5. Thus, one can conclude that in the nonautonomous system (1) there are periodic and quasiperiodic regimes at the small values of the pulsed amplitude and periodic regimes and chaos at the large values.

3. Conclusion

Thus by the example of the Dmitriev - Kislov generator the problem of synchronization by the periodic sequence of δ - function in the system with a three-dimensional phase space are considered. Different methods of the adding of external action to the system are studied. It is showed that the synchronization in the system with limit cycle embedded in a tree-dimensional phase space significantly depends on the direction of pulse action and on the attractor's form. If external pulses act in the plane, where the projection of phase portrait of autonomous system represents a circle or the object close to a circle, then synchronization picture in nonautonomous system will be closed to the synchronization picture typical for two-dimensional system with a circle limit cycle under the acting of the periodic sequence of δ - function. However, if projection of phase portrait essentially differ from the circle (for example, it is a figure-of-eight), then synchronization picture will be different. In the present paper it is showed that this synchronization picture will be differed both from the classical typical for the sinus-circle map and from the synchronization picture typical for two-dimensional system with a circle limit cycle under the action of the periodic sequence of δ - function.

Acknowledgements

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