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Features of Short-Pulse Synchronization of a Lorenz System

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Abstract—The pattern of synchronization of a Lorenz system by a periodic sequence of short pulses has been studied during the variation of a parameter determining the appearance of a chaotic attractor.

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The Lorenz system was among the first and most important models in nonlinear dynamics [1, 2]. The dynamics of this system has been studied in sufficient detail [2, 3] and is characterized by the appearance of a chaotic attractor known as the Lorenz type attractor. At present, much attention is devoted to the problem of synchronization in systems featuring dynamical chaos [4–7]. A very interesting problem in this respect is to trace the pattern of synchronization of a chaotic system during variation of a parameter determining the character of its autonomous dynamics.

This Letter presents the results of investigation of the behavior of a Lorenz system under the action of a periodic train of short pulses. It should be emphasized that the synchronization by short pulses can substantially differ from that in the case of a harmonic drive [8–10].

Let us consider a Lorenz system described by the following system of differential equations:

$$\begin{aligned} \dot{x} &= \sigma(y-x), \\ \dot{y} &= rx - y - xz, \\ \dot{z} &= -bz + xy + \varepsilon\sum_{n=-\infty}^{\infty}\delta(t-nT), \end{aligned} \qquad (1)$$

where x, y, and z are dynamical variables; σ, r, and b are the parameters; and ε and T are the amplitude and period of external pulses. In order to study the pattern of synchronization in system (1), we have plotted two-dimensional maps of dynamic regimes on the plane of parameters (amplitude ε versus frequency $\omega = 2\pi/T$) of the external action and constructed three-dimensional nonautonomous attractors in some regions of synchronization (Fig. 1). On these maps, white regions correspond to the regimes with period 1 (the period is determined in the corresponding Poincaré section), gray regions represent the regimes with period 2, and black regions correspond to chaotic dynamics. The system parameters σ and b were given fixed traditional values ($b = 8/3$, $\sigma = 10$), while parameter r was varied from zero to 28, which corresponds to the loss of stability in the autonomous Lorenz system (occurring at the origin of coordinates of the immobile point) and the appearance of a chaotic attractor. It should be noted that an autonomous system with a large value of this parameter (e.g., $r = 345$) is characterized by a stable limit cycle, and a decrease in r leads to the passage to chaos via a sequence of period doubling bifurcations.

As can be seen from Fig. 1a, the nonautonomous Lorenz system with a small value of r ($r = 5$) exhibits no very complicated regimes. Indeed, the most part of the plane of parameters is occupied by the regime with period 1. In this regime, a trajectory in the phase space arrives at an immobile point on the z axis between pulses, while each external pulse drives the system out of this point. However, external signals with small frequencies also make possible the regimes with period 2 and chaotic regimes, the regions of which are situated one above another in the vertical plane (Fig. 1a). These regions correspond to different numbers of turns of the limit cycle about unstable points of the system. As clearly seen in Fig. 1a, the attractor corresponding to the first structure makes one turn about an unstable point occurring in the region of positive x values. In the second region, the attractor exhibits an additional loop in the vicinity of a second unstable point. Analogous characteristic features in the structure of attractors of a nonautonomous Lorenz system are observed for the regions of regimes with periods 2 (Fig. 1a), 4, 8, etc. Moreover, it is clear that the regions of regimes with period 2 and chaos will also be observed for greater amplitudes of the external periodic action.

As the parameter r grows to $r = 15$, the regions of regimes with period 2 and chaos increase in size and shift toward greater values of the external signal fre-

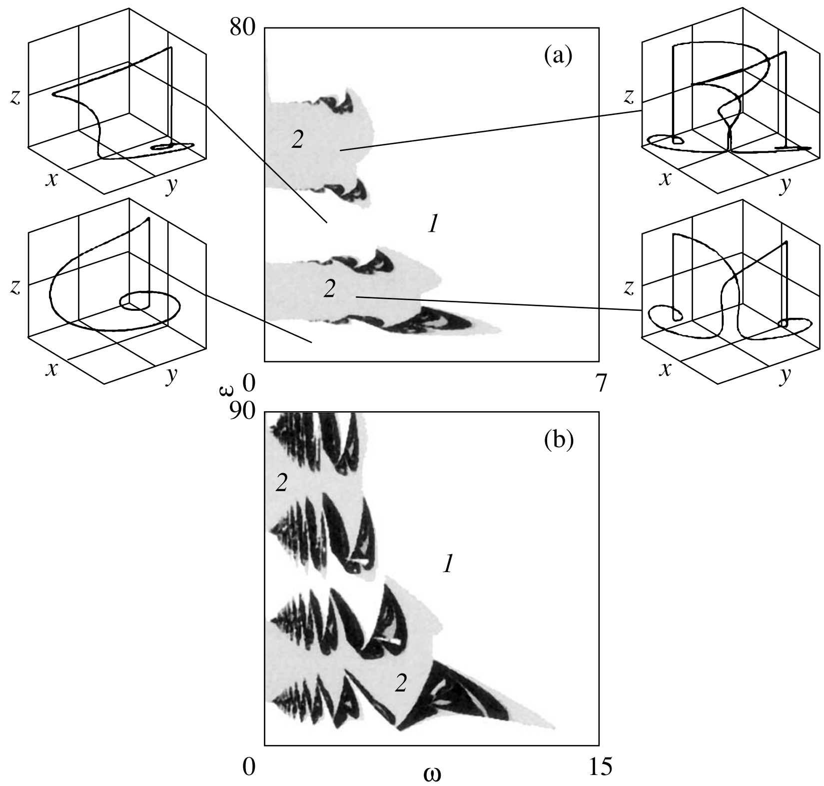

Fig. 1. Maps of dynamic regimes on the ε–ω plane of parameters and (insets) attractors of a nonautonomous Lorenz system synchronized by a periodic train of short pulses (δ functions); the maps correspond to Eqs. (1) with $b = 8/3$, $\sigma = 10$, and $r = 5$ (a) and 15 (b). Regions *1* (white) and *2* (gray) correspond to regimes with period 1 and 2, respectively; black regions correspond to chaotic dynamics.

quency (Fig. 1b). Moreover, the regions of chaotic dynamics become more clearly manifested, and the regions of both period 2 and chaos exhibit periodicity in the horizontal direction (especially pronounced for the structures arising at small values of the external pulse amplitude). In addition, the regions of period 2 and chaos are shifted slightly upward on the map of dynamic regimes, which implies that the same number of periodic structures is observed within a greater range of amplitudes.

The further increase in the r value (up to $r = 28$, which corresponds to the appearance of a chaotic attractor in the autonomous Lorenz system), leads to the appearance of two large regions with substantially different dynamics on the map of regimes (Fig. 2a). The first region corresponds to the top right part of the map, where the system still exhibits only the regimes with period 1, for which a trajectory in the phase space arrives at an immobile point on the z axis. The second region occupies the bottom left part of the map, which corresponds to the regimes of synchronization with various periods and quasi-periodic regimes. Note that the alternating regions of synchronization exhibit periodicity not only in the vertical plane but in the horizontal direction as well. Similar to the case of smaller r (Fig. 1a), attractors belonging to various vertical structures with the same period differ by making additional loops about one of the immobile points (Fig. 2a). For example, an attractor constructed for the lowest region of synchronization with period 2 (i.e., the top right attractor in Fig. 2a) makes one turn about each of the two unstable immobile points of the corresponding autonomous Lorenz system. An attractor with the same period constructed for the next to bottom region of synchronization with period 2 (i.e., the top left attractor in Fig. 2a) has an additional loop in the vicinity of the left unstable immobile point. Attractors with the same period constructed for synchronization regions alternating in the horizontal plane exhibit some other characteristic features. All these attractors also make turns about unstable immobile points of the autonomous Lorenz system, but the number of turns depends on the particular synchronization region, increasing on the passage from right to left in the map of dynamic regimes (Fig. 2a).

Finally, let us briefly consider the pattern of synchronization observed in system (1) with a very large value of parameter r, for example, $r = 345$. An autonomous Lorenz system with this parameter r has a stable limit cycle of period 1, while a decrease in r leads to the passage to chaos via a sequence of period doubling bifurcations based on this limit cycle. Nevertheless, the

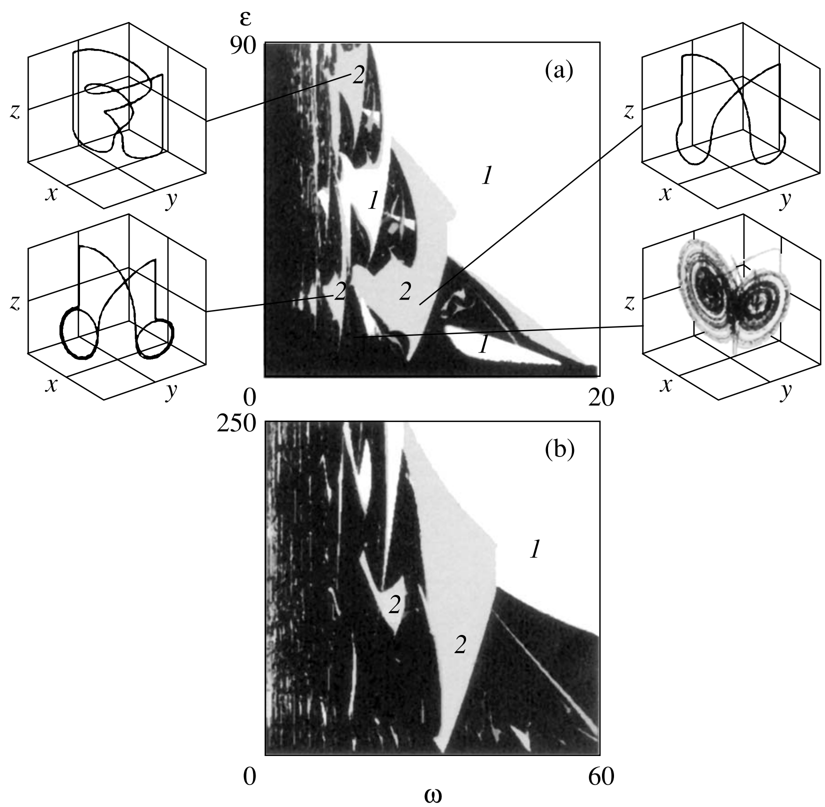

Fig. 2. Maps of dynamic regimes on the ε–ω plane of parameters and (insets) attractors of a nonautonomous Lorenz system synchronized by a periodic train of short pulses (δ functions); the maps correspond to Eqs. (1) with $b = 8/3$, $\sigma = 10$, and $r = 28$ (a) and 345 (b). Regions *1* (white) and *2* (gray) correspond to regimes with period 1 and 2 respectively; black regions correspond to chaotic dynamics.

dynamics of the nonautonomous Lorenz system resembles that observed for the values of r corresponding to a chaotic attractor (Figs. 2a and 2b). Indeed, the plane of parameters ε versus ω can also be divided into two regions, one corresponding to the regimes with period 1 and the other representing various periodic and quasi-periodic regimes. However, the region of regimes with period 1, as well as that of quasi-periodic structures, is observed only for very large amplitudes ($\varepsilon > 100$) and/or frequencies ($\omega > 30$) of external pulses (Fig. 2b). At the same time, the structure of synchronization regions in the bottom part of the map is quite different, representing rather narrow tongs of synchronization with various periods surrounded by regions where system (1) exhibits quasi-periodic behavior. Therefore, the character of synchronization in Lorenz systems (1) with large values of parameter r is a similar to the classical pattern. Note also that, for large values of parameter r, the map of dynamic regimes reveals no clear periodicity in the arrangement of synchronization regions (Fig. 2b).

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