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Hyperbolic Chaos Generator Based on Coupled Drift Klystrons

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Abstract—A scheme of the generator of chaotic oscillations in the microwave band is proposed that is based on two drift klystrons coupled in a ring circuit. The first klystron doubles the frequency of the input signal and the second klystron mixes the second harmonic signal and the reference signal representing a periodic sequence of radio pulses filled with the third harmonic, followed by separation of the differential first harmonic signal. As a result, the transformation of the signal phase during the period of pulse repetition is described by a stretching map of the circle (Bernoulli map) and demonstrates chaotic behavior. Results of numerical calculations are presented that confirm the implementation of the proposed mechanism to ensure the generation of robust, structurally stable chaos in the system under consideration.

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Robust, hyperbolic chaos is considered to be the strongest type of chaotic behavior, in which case the strange attractor does not contain stable periodic orbits and only comprises trajectories of the saddle (hyperbolic) type [1]. These attractors are characterized by robustness or so-called structural stability, which implies that the character of the system dynamics and the attractor structure are insensitive with respect to variations of the parameters and functions describing the system. Recently, it was suggested that physical systems be constructed with hyperbolic-type attractors, and an example of the device possessing such an attractor was presented [2, 3]. The proposed system consists of two coupled oscillators operating at base and double frequencies, which alternatively become active and transfer the excitation to each other so that the transformation of the oscillation phase over a compete cycle is described by the stretching map of a circle or the Bernoulli map. It is of interest to extend this approach to distributed wave systems with an infinite number of the degrees of freedom. As is known, a harmonic wave propagating in a nonlinear medium becomes rich in harmonics. Thus, various spectral components could play the role of coupled oscillators in the lumped subsystems. We suggest that the implementation of this idea offers a possibility to generate robust, structurally stable chaos at higher frequencies, in particular in the microwave and optical ranges.

In this Letter, we propose a scheme of a microwave generator of chaotic oscillations, which comprises two drift klystrons connected in a closed ring (Fig. 1). In the first klystron, the input cavity is tuned to frequency ω, while the output cavity is tuned to the second harmonic 2ω, so that this klystron operates as a frequency multiplier. The signal from the output cavity of the first klystron is fed to the input cavity of the second klystron via a transmission line containing an attenuator and a phase shifter, which allow the signal phase and amplitude to be adjusted. In the second klystron, this input signal is mixed with a reference sig-

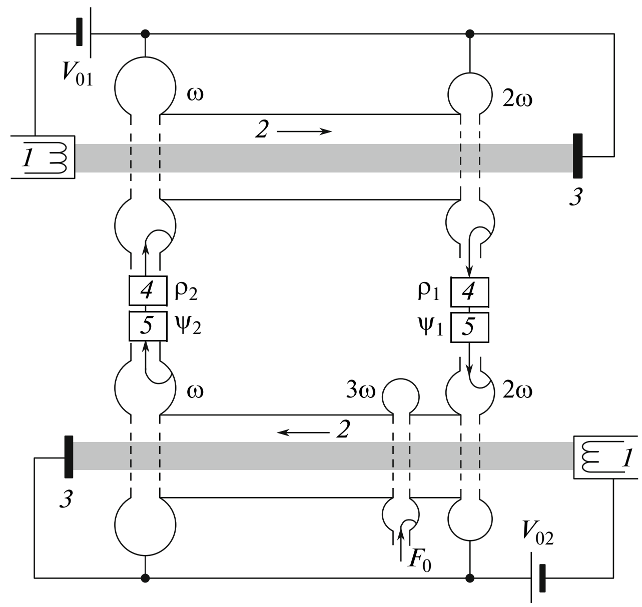

Fig. 1. Schematic diagram of the proposed chaos generator based on coupled drift klystrons: (*1*) electron guns; (*2*) electron beams; (*3*) collectors; (*4*) attenuators; (*5*) phase shifters.

nal, which represents a sequence of radio pulses with carrier frequency 3ω. In the output cavity of the second klystron, a signal with a difference frequency of ω is separated and fed to the input cavity of the first klystron, thus closing the feedback circuit.

It should be noted that the generators of chaotic oscillations based on coupled klystrons were also studied previously [4, 5] and it was established that these devices offer some advantages. However, the consideration in [4, 5] was restricted to systems with cavities tuned to the base frequency. It can be expected that the proposed scheme will retain the advantages of systems studied previously and, in addition, will possess the practically valuable property of structural stability.

A mathematical model of the proposed generator is constructed by analogy with other systems based on drift klystrons with time delay feedback [5, 6]. According to this approach, amplitudes of the corresponding harmonics of the beam current are calculated and substituted into nonstationary equations that describe the excitation of cavities with allowance for their coupling. For the sake of simplicity, we assume that the system parameters (Q values and natural frequencies of cavities, electron beam currents, accelerating voltages) of the two klystrons are identical. As a result, we arrive at the following system of dimensionless equations (details of the derivation will be presented in a separate publication):

$$\dot{F}_1^{\omega} + F_1^{\omega} = \rho_2\mu^{\omega}e^{i\psi_2}F_2^{\omega}, \tag{1}$$

$$\dot{F}_1^{2\omega} + \delta F_1^{2\omega} = 4\alpha_1 J_2(2|F_1^{\omega}(t-\tau)|)e^{2i(\varphi_1^{\omega}(t-\tau)-\theta_0)}, \tag{2}$$

$$\dot{F}_2^{2\omega} + \delta F_2^{2\omega} = \rho_1\mu^{2\omega}e^{i\psi_1}F_1^{2\omega}, \tag{3}$$

$$\begin{aligned}\dot{F}_2^{\omega} &+ F_2^{\omega}\\ = 2\alpha_2 e^{-i\theta_0}s &\sum_{m=-\infty}^{\infty} i^m J_{3m+1}(|F_2^{2\omega}(t-\tau)|)J_{2m+1}(F_0)\\ &\times e^{-i(3m+1)\varphi_2^{2\omega}(t-\tau)},\end{aligned} \tag{4}$$

where the subscripts 1, 2 indicate the klystron numbers, while the superscripts ω, 2ω specify the resonance frequency of a cavity; quantities F_j^k are the normalized complex amplitudes of fields in the corresponding cavities, with the phases defined as $\varphi_j^k = \arg(F_j^k)$; $F_0(t)$ is a function determining the normalized amplitude of the third harmonic signal as a function of the dimensionless time t; parameter δ is the ratio of Q values of the cavities operating at frequencies ω and 2ω; ρ_j and ψ_j are the attenuation coefficients and phase shifts, respectively, in the transmission lines; μ^k are the coupling coefficients, which depend on the Q values of the corresponding cavities; α_j are the excitation parameters; θ_0 is the unperturbed electron transit angle in the drift space; and J_n are nth order Bessel functions of the first kind. The right-hand parts of Eqs. (2) and (4) contain terms with delayed argument, the appearance of which is due to a finite time τ of electron beam propagation in the drift space. The oscillator dynamics is significantly influenced by the excitation parameters α_j, which can be considered as the normalized electron beam currents (see [5, 6]). Although the two klystrons are assumed to be identical, the α_j values in the general case are different. Indeed, the modulation in the first klystron is performed by the cavity operating at the fundamental frequency, while that in the second klystron it is performed by the cavity operating at the second harmonic, while the α_j values depend on the parameters of a modulating cavity.

Let the reference third harmonic signal be supplied from an external source in the form of a sequence of rectangular pulses with a repetition period equal to the time of signal passage via the feedback circuit. Let us assume that field oscillations in the cavities build up quite rapidly and consider the evolution of variables for a discrete series of time moments $t_n = 2n\tau$. After some transformations, the system of equations (1)–(4) can be reduced to the following iterative map:

$$\begin{aligned}F_{n+1}e^{i\varphi_{n+1}} &= 2\alpha_2\rho_2\mu^{\omega}e^{i(\psi_2-\theta_0)}\\ \times\sum_{m=-\infty}^{\infty} &i^m J_{3m+1}\left(\frac{4\alpha_1\rho_1\mu^{2\omega}}{\delta^2}J_2(2F_n)\right)J_{2m+1}(F)_0\\ &\times e^{-i(3m+1)(2(\theta_0-\varphi_n)+\psi_1)},\end{aligned} \tag{5}$$

where $F_n = |F_1^{\omega}(t_n)|$ and $\varphi_n = \arg(F_1^{\omega}(t_n))$. In this expression, the right-hand side represents an infinite series.

Let us consider the case of a strong attenuation ($\rho_1 \ll 1$) of the second harmonic signal supplied from the output of the first klystron to the input of the second klystron. The main contribution is due to the term with $m = 0$ containing the product of the first-order Bessel functions, while the contribution from other members of this series (containing products of the Bessel functions of higher orders) can be ignored. Then, Eq. (5) simplifies and acquires the following form:

$$\begin{aligned}F_{n+1}e^{i\varphi_{n+1}} &= 2\alpha_2\rho_2\mu^{\omega}J_1(F_0)J_1\left(\frac{4\alpha_1\rho_1\mu^{2\omega}}{\delta^2}J_2(2F_n)\right)\\ &\times e^{i(\psi_2-\theta_0+2(\varphi_n-\theta_0)+\psi_1)}.\end{aligned} \tag{6}$$

As can be seen from this formula, the transformation of the field phase can be approximately described by the map $\varphi_{n+1} = 2\varphi_n + \text{const}(\text{mod}2\pi)$ This is a stretching map of the circle or the Bernoulli map that demonstrates chaos with a Lyapunov exponent of $\Lambda = \ln 2 \approx 0.693$.

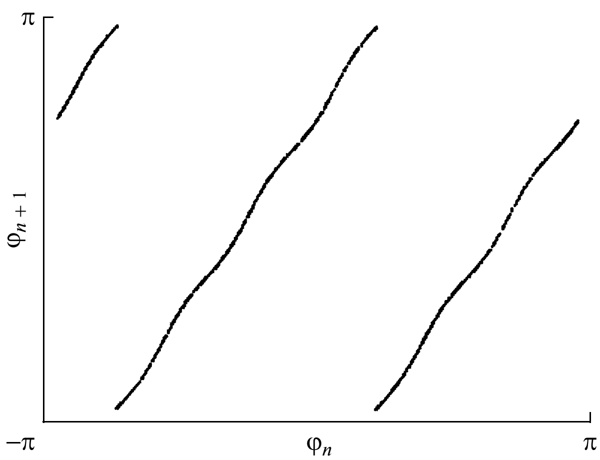

Fig. 2. Iterative diagram of transformation of the phase of oscillations (see text for explanations)

Since the chaotic dynamics of the Bernoulli map is robust, we may expect that a chaotic regime of the same kind will be observed in map (5), at least at not very large values of the attenuation coefficient ρ_1. This assumption is confirmed by the results of numerical calculations. For the sake of simplicity, we also assume that $\alpha_1 = \alpha_2 = \alpha$ (which can always be achieved by adjusting the parameters).

Figure 2 shows an iterative diagram for the phase φ_n, which was calculated for the following values of parameters: $\alpha = 15.5$, $\delta = 2$, $F_0 = 1.84$, $\mu^{\omega} = \mu^{2\omega} = 1$, $\theta_0 = 500$, $\psi_{1,2} = 0$, $\rho_2 = 0.5$, and $\rho_1 = 0.05$. As can be seen from this diagram, the phase transformation is actually described by the map with a topology analogous to that of the Bernoulli map; the results of modeling weakly depend on the number of terms retained in series in the right-hand part of Eq. (5) (results presented in Fig. 2 were obtained with allowance for 11 terms).

Figure 3 shows a portrait of the attractor in projection onto the plane of complex variable F_n. As can be seen, the attractor has a topology of the Smale–Williams type, which is typical of the systems with hyperbolic chaos [1–3].

Figure 4 presents the dependence of the highest Lyapunov exponent Λ on the excitation parameter α for fixed values of the other parameters. At $\alpha \le 14.3$, the only attractor of map (5) is a stable point at the origin. In the interval of $14.3 \le \alpha \le 17.8$, this point coexists with a chaotic attractor, for which Λ is constant and approximately equal to ln2, which is evidence of a hyperbolic chaos. At $\alpha \approx 17.8$, the hyperbolic attractor exhibits crisis, in which the attractor touches a boundary of the basin of attraction (see [7]). For even greater values of α, the only attractor is again the fixed point at the origin of coordinates.

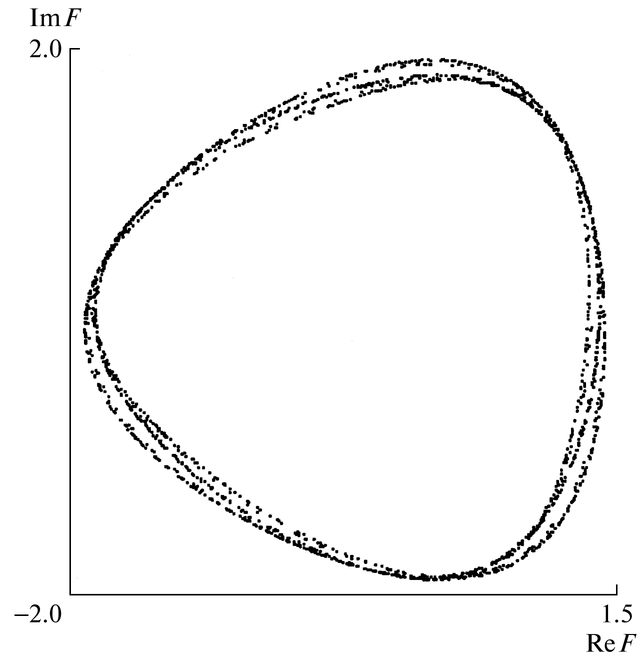

Fig. 3. Projection of the attractor of map (5) onto the ReF_n–ImF_n plane.

In conclusion, we have proposed a scheme of the generator of chaotic oscillations in the microwave band, which is based on a system of two coupled drift klystrons. A mathematical model of this generator is constructed that reduces to map (5) under the assumption of the instantaneous build-up of oscillations in the cavities of klystrons. The results of numerical calculations show that, for certain values of parameters, a chaotic regime is established in the system, according to which the transformation of the phase of oscillations during the pulse repetition period is described by a stretching map of the circle. It should

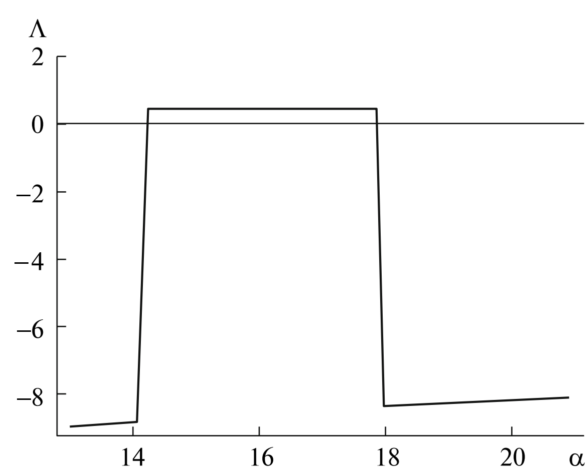

Fig. 4. Plot of the highest Lyapunov exponent Λ of map (5) versus excitation parameter α.

be noted that the simulations using the initial differential system (1)–(4) with time delay yield qualitatively analogous results. A detailed comparison of the two models will be presented in a separate publication.

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