CHAOTIC AND QUASIPERIODIC OSCILLATIONS IN RADIOPHYSICAL GENERATORS

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Annotation. In the present paper in numerical and radiophysical experiments formation of multi-modes chaotic attractor, based on the multi-dimensional quasiperiodic torus is observed.

Keywords. Dynamical systems, quasiperiodic oscillations, chaos, Lyapunov exponents.

One of the promising direction of practical application of radiophysical generators of chaotic signals is to use them for secure communication systems [1-3]. This direction has been developing for quite a long time, however, there are a number of problems, not allowing to use these generators in applications. Such kind problems include: the dependence of generation mode of chaotic signal from the parameters and initial conditions, sensitivity to noise, the problem of confidentiality of transmitted information, e.t.c. In general all this problems connected with different properties of chaotic attractors.

Investigation of multi-frequency quasi-periodic oscillations and transition from quasiperiodicity to chaos are actual and important problem in different areas of science [4-5], including systems of secure communication. Recently in [6] was suggested the model of five globally coupled oscillators of van der Pol type, in which Landau-Hopf scenario was observed: the sequence of birth of torus of high dimension. In [7] was suggested a new methodic of construction of multi-fold Poincare section for observation of multi-dimensional tori. In the present paper the dynamic of multi-contour self-generators, consisting of five independent oscillating systems with common controlling circuit of excitation of each oscillating mode. In the present paper we would like to present numerical and experimental investigation of the systems, which are represented system of coupled self-generators and system of multi-contours self-generator with common controlling circuit.

The starting point of our investigation was the dynamics of five coupled van der Pol oscillators (Fig.1). Each oscillator is characterized by two phase variables and one frequency of own oscillations. Therefore, we deal with ten-dimensional system and with five incommensurable frequencies. In Fig. 1 one can see the scheme of electronic circuit of five generator of van der Pol type. Each generator consists of an oscillatory circuit formed by inductors $L_{1\text{-}5}$ and capacitors $C_{1\text{-}5}$, and of nonlinear elements represented by back-to-parallel linked semiconductor diodes (D_1-D_2). Oscillators are differ only by frequencies. All another parameters is identical for each oscillator. Frequency mismatch between frequencies of oscillators was realized by different capacity of capacitors $C_{1\text{-}5}$. Excitation of the self-oscillations occurs due to the negative resistance blocks ($R_{1\text{-}5}$) assembled on the operational amplifiers ($OA_{1\text{-}5}$). R_2 and R_3 represent negative feedback circuit of operational amplifier OA_1. Varying resistance of the resistor R_2 we can change coefficient of amplifier. The dynamical variables are voltage outputs from operational amplifiers ($OA_{1\text{-}5}$), and its derivative, formed by derivative amplifiers ($OA_{6\text{-}10}$). The coupling between the generators is provided by variable resistor $R_{i\text{-}i+1}$, $i = (1 \div 5)$, connecting identical points of the two circuits, and we consider the system with ring coupling, $R_{5\text{-}6}=R_{5\text{-}1}$ (Fig. 1). The strength of coupling is inversely proportional to the resistor. Dynamical variables X, X', Y, Z, W, V input to IO device and variables Y', Z', W', V' input to gate circuit. At small coupling the system demonstrates five-frequency quasi-periodic oscillations. With increasing of coupling the one of the frequencies will be locked and we can observe transition from five-frequency torus to the four-frequency torus and the three-frequency torus. In order to distinguish quasi periodic oscillations with different number of frequency components we apply technique of visualization of fold Poincare section. For visualization of invariant curve of five-frequency quasi-periodic oscillation we should realize four-fold Poincare section. In order to escape situation which occurred in the case of two coupled generators of quasi-periodic oscillations we will analyze dynamical variables of the one of oscillators (*X*, *X'*) and one of variables of another oscillators (*Y*, *Z*, *W*, *V*), and realize Poincare sections by derivatives of another variables (*Y'*, *Z'*, *W'*, *V'*) using gate circuit [7].

In Fig. 2, transformation of phase portraits corresponding to this bifurcation is shown. In Fig. 2, phase portraits are indicated by red color (colored on-line) and invariant curves in double Poincare section are indicated by blue color. Figure 2a corresponds to the situation before bifurcation and Fig. 2c corresponds to the situation after bifurcation. As one can see, invariant curve very-well demonstrates such kind of transformation. In Fig.2b phase portrait before doubling is depicted.

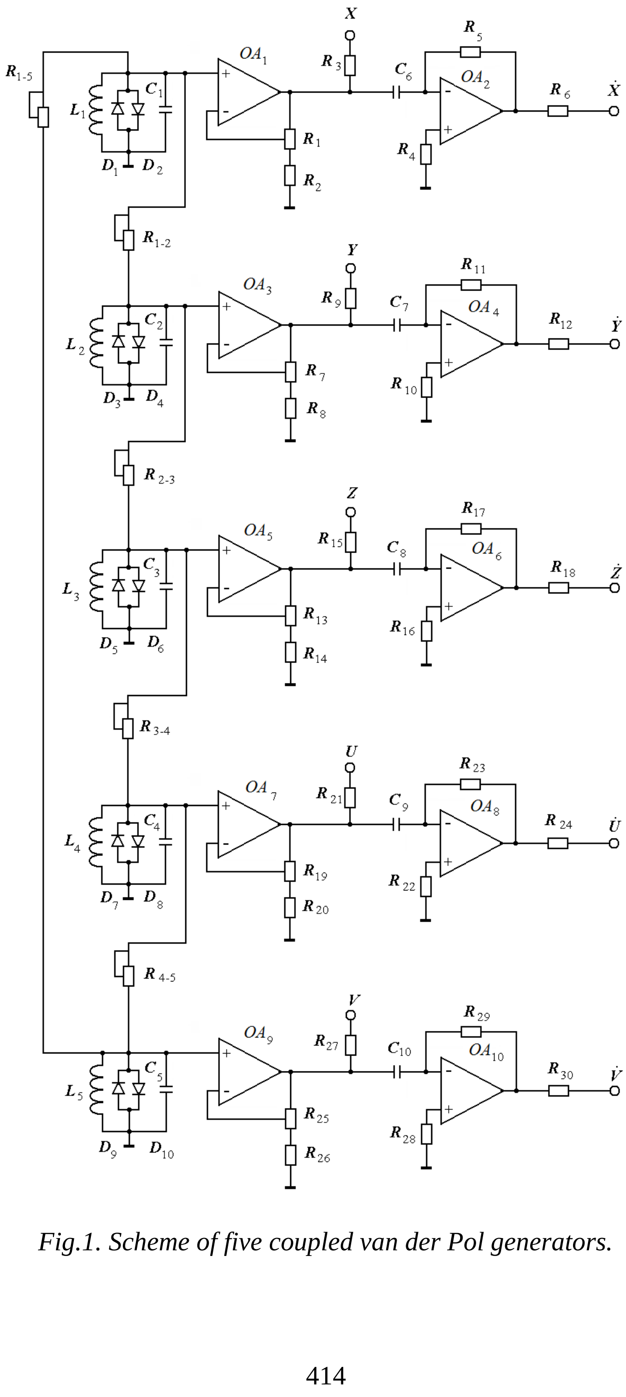

Fig.1. Scheme of five coupled van der Pol generators.

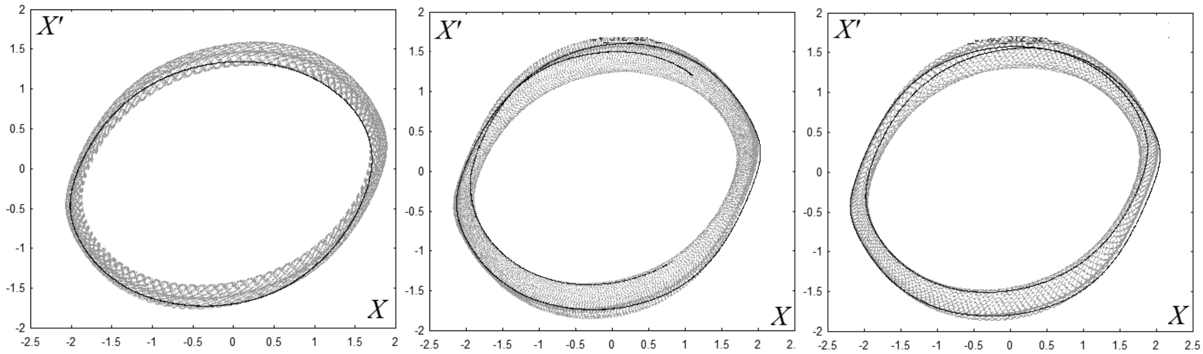

Fig.2. Doubling of three-frequency tori in the system of five coupled generators of van der Pol type. Gray color corresponds to phase portraits in projection (X, X'), black color corresponds to phase portraits in double Poincare section by hyper-surface Y' = 0, V' = 0.

Then let us turn to study the dynamic of multi-contour self-generators, consisting of five independent oscillating systems with common controlling circuit of excitation of each oscillating mode. On Fig.3 the scheme of multi-contour circuit is show. The base of multi-contour self-generator are five oscillatory L_1C_1-L_5C_5 circuits. These circuits determine frequencies of the five base oscillatory modes. In experiments all frequencies were distributed in the next way: f_1 = 6.65 kHz, f_2 = 11.53 kHz, f_3 = 21.38 kHz, f_4 = 41.88 kHz, f_5 = 82.24 kHz. As active element we use operational amplifiers OA_1-OA_5 with controllable coefficients of amplification of voltage, which work in linear mode and were used as junction of oscillatory contours between itself and controlling of excitation of each mode of self-generator. In experiment we can change amplification of each mode, and its were marked by letters: k_1, k_2, k_3, k_4, k_5. Amplifier OA_{11} is a summing, inverting the nonlinear amplifier, providing nonlinear limitation of amplitude of oscillations. Amplifier OA_{12} is a invertor amplifier working in linear mode, providing additional amplification of summing signal. Amplifiers OA_6-OA_{10} are differential amplifiers, which are used for getting of derivative of voltage from outputs OA_1-OA_5. Controlling parameters of studying system were coefficients of amplification k_1 - k_5 of operational amplifiers OA_1 - OA_5.

In the case when all coefficients of amplifications equals zero ($k_1 = 0$, $k_2 = 0$, $k_3 = 0$, $k_4 = 0$, $k_5 = 0$) there is no excitation of oscillations. If we increase one of the coefficients, self-oscillations on the mode of the first oscillatory contours occur. In Fig.4 a one can see limit cycle which occur at $k_1 = 3$ ($k_2 = k_3 = k_4 = k_5 = 0$). In Fig.4 two-dimensional projections of phase portraits on the plane (x, Σ) are depicted, where $\Sigma=x+y+z+u+v$. Then we added another mode and at $k_1 = 3$, $k_2 = 7$, $k_3 = k_4 = k_5 = 0$ we observed excitation of the second oscillatory mode, and in Fig.4b one can see phase portrait in the form of two-dimensional torus. Then we added the third mode, and at $k_1 = 3$, $k_2 = 7$, $k_3 = 7$, $k_4 = k_5 = 0$. we observe three-frequency torus. In Fig.3c, d two-dimensional projection of phase portrait and Poincare section by hypersurface $Y' = 0$ are depicted. For the three-frequency torus in experiment was observed resonance two-frequency torus. In Fig.4e, f two-dimensional projection of phase portrait and Poincare section by hypersurface $Y' = 0$ for resonance at $k_1 = 10$, $k_2 = 10$, $k_3 = 10$, $k_4 = k_5 = 0$ are shown.

Thus, in the physical experiment the system of multi-contours self-generators with common controlling circuit was studied. For this system we observed gradual occurrence of two-, three-, four-, and five-frequency oscillations and transition to chaos on the base of multi-dimensional torus.

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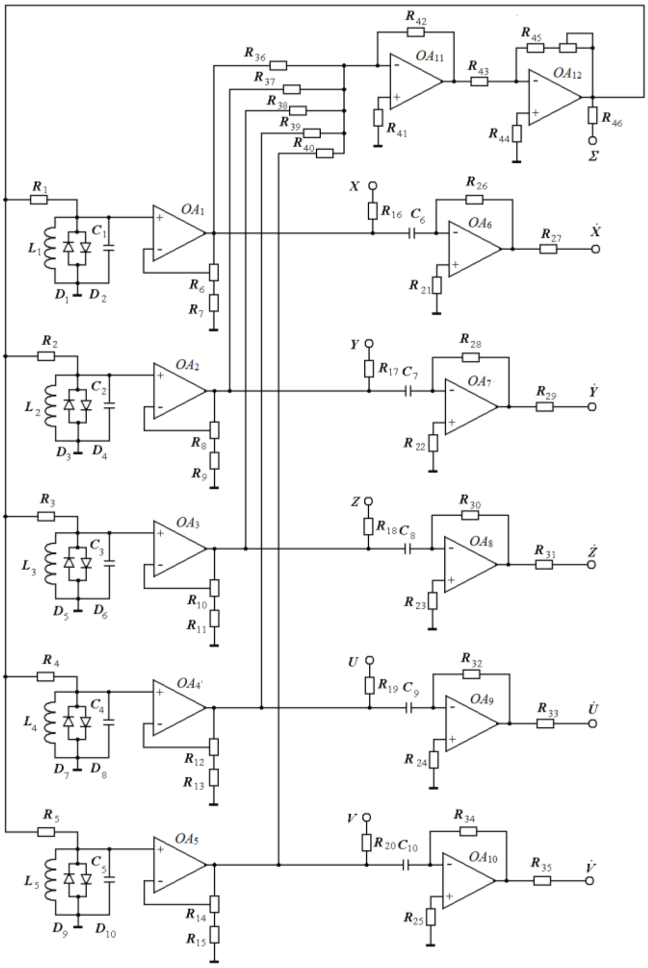

Fig. 3. Scheme of electronic circuit of five-contours self generator with common controlling circuit.

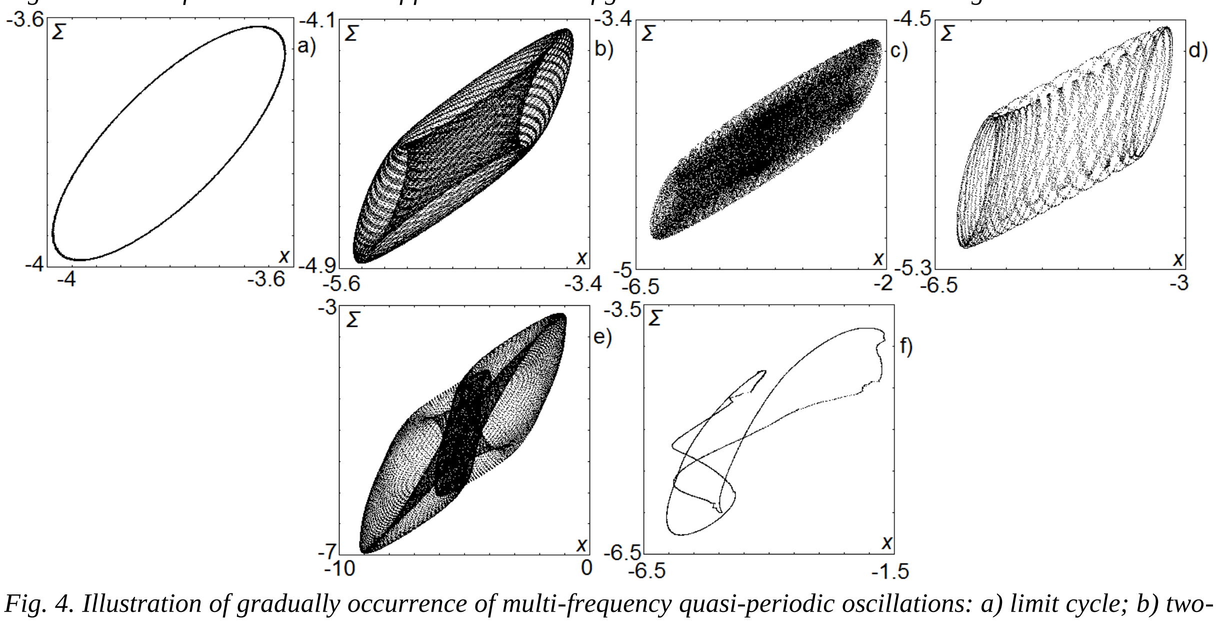

Fig. 4. Illustration of gradually occurrence of multi-frequency quasi-periodic oscillations: a) limit cycle; b) two-dimensional torus; c), d) three-frequency torus, phase portrait and Poincare section; e), f) two-frequency resonance torus, phase portrait and Poincare section.

Refernce

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