

Cooperative maintenance of cellular identity in systems with intercellular communication defects

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(Dated: 2 January 2020)

The cooperative dynamics of cellular populations emerging from the underlying interactions determines cellular functions, and thereby their identity in tissues. Global deviations from this dynamics on the other hand reflects pathological conditions. However, how these conditions are stabilized from dysregulation on the level of the single entities is still unclear. Here we tackle this question using generic Hodgkin-Huxley type of models that describe physiological bursting dynamics of pancreatic β -cells, and introduce channel dysfunction to mimic pathological silent dynamics. The probability for pathological behavior in β -cell populations is $\sim 100\%$ when all cells have these defects, despite the negligible size of the silent state basin of attraction for single cells. In stark contrast, in a more realistic scenario for a mixed population, stabilization of the pathological state depends on the size of the sub-population which acquired the defects. However, the probability to exhibit stable pathological dynamics in this case is less than $\sim 10\%$. These results therefore suggest that the physiological bursting dynamics of a population of β -cells is cooperatively maintained, even under intercellular communication defects induced by dysfunctional channels of single-cells.

Keywords: Beta-cells dynamics; dysfunctional channels; heterogeneous cell population

Emergence of undesired dynamics in systems of interacting cells is often associated with pathological conditions. However, how such conditions can be stabilized on a population level from dysregulation on the level of single cells is unclear. Using generic Hodgkin-Huxley formalism to describes physiological bursting activity of pancreatic β -cells, we study here how a dysregulation on a single cell level can dominate the dynamics of the cellular population. The dysregulation is mimicked by introducing a potassium-like ion channel with decreased opening probability due to non-monotonic voltage dependence, that subsequently stabilizes a silent state in addition to the bursting activity. We found that when all of the cells have acquired channel defects, a coupling range exists for which the silent pathological state is a dominant dynamical behavior of the system, despite the small basin of attraction of the silent state on the single cell level. This holds true even for heterogeneous populations, when a sub-population does not manifest the silent behavior. In contrast, the bursting dynamics of a mixed population in which only a portion of cells can exhibit bistability between silent and bursting dynamics and are subsequently poised in the silent state, can be only disrupted if at least $N/2 + 1$ cells have the channel dysfunction. The probability for the full population to exhibit silent dynamics in this case is however less than 10%. These results thereby demonstrate that the bursting dynamics of β -like cells is cooperatively maintained, even

when a large sub-fraction of this population acquired dysfunctional channels that induce intercellular communication defects.

I. INTRODUCTION

The emergent properties of complex biological systems are largely determined by the interactions of the elements that constitute these systems. Synchronous firing of ensembles of interacting neurons enables for example, complex collective tasks such as Hebbian learning and memory^{1,2} or wake-sleep cycles^{3,4} to be executed. On the other hand, the collective behavior of homogeneous or heterogeneous cell types including their arrangement and types of contacts determines specific functions of tissues and organs. For instance, it has been experimentally observed that the well-synchronised electrical activity (spiking and bursting) of primary β -cells in the pancreatic islet is what drives insulin secretion⁵⁻⁷, where the secretion increases with the fraction of time that the cells spend in the spiking state. The duration of the silent phase between two bursts is in this case regulated by the rate at which calcium is removed from the cell interior. The spiking oscillations typically display a period of 1-10s, whereas the duration of the bursting period varies from 0.2 to 5min. In contrast, although disassociated cells show increased electrical activity at elevated glucose concentrations, this activity is variable from cell to cell, with some cells being electrically “silent” and others continuously “spiking” at any given glucose concentration⁸⁻¹². Thus, synchronization of electrical activity via gap junctions has long been argued as essential for normal glucose-dependent insulin secretion in the intact islet^{13,14}. Deviation from the characteristic dynamical behav-

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ior of the system on the other hand reflects pathological conditions. How such global deviations in the dynamics are stabilized from dysregulation on the level of the interacting elements is however unclear. This question becomes even more prominent considering that it has been experimentally demonstrated that in transgenic mice, the collective behavior of β -cells populations where approximately 70% of the cells express zero functional potassium channels can maintain strong glucose dependence of metabolism, Ca, membrane potential, and insulin secretion from intact islets, although the glucose dependence is lost in isolated transgenic cells¹⁵.

Here we use numerical simulations to tackle how a change in the global dynamics of a coupled system can be established by dysregulation on a sub-population level. To address this, we employed a generic Hodgkin-Huxley formalism which on a single cell level describes physiological bursting activity of pancreatic β -cells^{16–18}. Additionally, we also use a modified version of this model where a potassium-like ion channel with decreased opening probability is introduced¹⁹ to stabilize a silent state in addition to the bursting activity, thereby enabling bistability on a single-cell level to be established. This silent state mimics ion channel dysfunction in a form of modified opening probability due to non-monotonic voltage dependence. We use these models to investigate how deviation from the physiological dynamics of the population of coupled β -cells is triggered depending on the number of cells acquiring dysregulation in channel activity. Three cases are therefore analyzed: (1) dynamics of two coupled β -cells under physiological conditions (bursting activity), (2) dynamical behavior of two coupled cells that have acquired the dysfunction (stable steady state), but one or both of them do not manifest it in isolated cells (heterogeneous population) and (3) dependence of the global population dynamics on number of cells with channel defects (mixed population). We found that the dynamics of a population of two coupled cells, each characterized with bistability between silent and bursting state but being differentially poised in the stable attractors (heterogeneous case), can be dominated by the silent pathological state for an intermediate coupling range, despite the small basin of attraction of the silent state on a single cell level. In contrast, the bursting dynamics of a mixed population in which only a portion of cells can exhibit bistability between silent and bursting dynamics and are subsequently poised in the silent state, can be only disrupted if at least $N/2 + 1$ cells have the channel dysfunction. The probability for the full population to exhibit silent dynamics in this case is however less than 10%. These results thereby demonstrate that the identity of a mixed population of β -like cells reflected through their bursting dynamics is cooperatively maintained, even when a large sub-fraction of this population acquired dysfunctional channels that induce intercellular communication defects.

II. RESULTS

A. Single β -cell models of bursting and silent dynamics

The electrical activity of pancreatic β -cells relies on multiple types of voltage- and ligand-gated ion channels that are permeable to inorganic ions such as sodium, potassium, chloride and calcium^{5,20,21}, and regulate membrane potential, ion homeostasis and electrical signaling²².

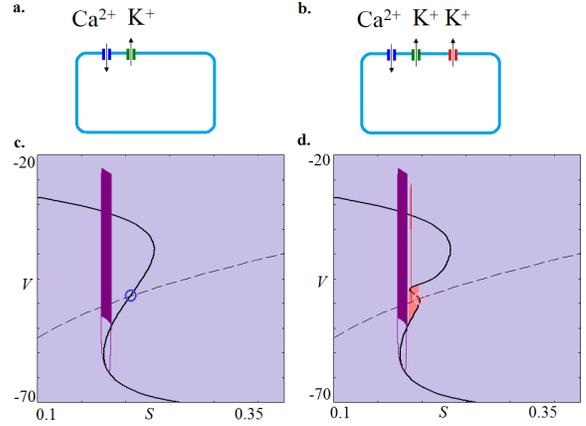


FIG. 1. **Dynamical structure of the β -cell models (Eqs. 1-7).** Schematic representation of a β -cell in **a.** physiological conditions (blue: Ca-channel, green: K-channel with physiological opening probability) and **b.** with an additional potassium channel with decreased opening probability (red). Fast and slow manifolds (black solid and dashed lines, respectively), attractors and their basins of attraction (blue: unstable node, red: stable node, violet: bursting attractor) are shown for **c.** $k^{(i)} = 0$, corresponding to Fig. 1a and **d.** $k^{(i)} = 1$, Fig. 1b. Parameters: $\tau = 0.02$, $\tau_S = 35$, $\sigma = 0.93$, $g_{Ca} = 3.6$, $g_K = 10.0$, $g_{K2} = 0.2$, $g_S = 4.0$, $V_{Ca} = 25.0$, $V_K = -75.0$, $V_m = -20.0$, $\theta_m = 12.0$, $V_n = -16.0$, $\theta_n = 5.6$, $V_S = -35$, $\theta_S = 10.0$, $V_p = -47.0$, $\theta_p = 1.0$.

To depict the bursting dynamics of pancreatic β -cells as observed under physiological conditions, as well as to simultaneously model silent dynamics caused upon ion channel dysregulation, we use a model based on the Hodgkin-Huxley formalism^{17,18} with modifications proposed by Stankevich and Mosekilde¹⁹:

$$\begin{aligned} \tau \dot{V}^{(i)} &= -I_{Ca}(V^{(i)}) - I_K(V^{(i)}, n^{(i)}) - k^{(i)} I_{K2}(V^{(i)}) \\ &\quad - I_S(V^{(i)}, S^{(i)}) \\ \tau \dot{n}^{(i)} &= \sigma (n_\infty(V^{(i)}) - n^{(i)}) \\ \tau_S \dot{S}^{(i)} &= S_\infty(V^{(i)}) - S^{(i)} \end{aligned} \quad (1)$$

where $V^{(i)}$ represents the membrane potential of the i -th cell, the functions $I_{Ca}(V^{(i)})$, $I_K(V^{(i)}, n^{(i)})$, $I_S(V^{(i)}, S^{(i)})$ define the three intrinsic currents, fast calcium, potassium and slow potassium respectively, such that:

$$I_{C_a}(V^{(i)}) = g_{C_a} m_{\infty}(V^{(i)})(V^{(i)} - V_{C_a}) \quad (2)$$

$$I_K(V^{(i)}, n^{(i)}) = g_K n(V^{(i)} - V_K) \quad (3)$$

$$I_S(V^{(i)}, S^{(i)}) = g_S S(V^{(i)} - V_K) \quad (4)$$

The gating variables for $n^{(i)}$ and $S^{(i)}$ are the opening probabilities of the fast and slow potassium currents:

$$\omega_{\infty}(V^{(i)}) = [1 + \exp \frac{V_{\omega} - V^{(i)}}{\Theta_{\omega}}]^{-1}, \omega = m, n, S \quad (5)$$

The function $I_{K2}(V^{(i)})$ on the other hand defines an additional voltage-dependent potassium current. It has been previously demonstrated that a single oscillator, in absence of I_{K2} (when $k^{(i)} = 0$, Fig. 1a), is characterized with a bursting attractor that is born via a Hopf bifurcation for $V_S = -44.7mV$. At this parameter value, the equilibrium point loses its stability^{23,24}. Although the bursting attractor is born in the vicinity of the equilibrium point, this point moves away from the bursting attractor for increase in V_S . Even more, the two dimensional projection of the phase portrait together with the fast and slow manifolds when $V_S = -35mV$ (Fig. 1c) demonstrates that the periodic trajectories do not intersect the neighborhood of the steady state and the bursting attractor terminates in a homoclinic bifurcation as the trajectory hits the slow manifold^{23,24}. Calculating the eigenvalues in this case shows that the equilibrium point is an unstable node $(\lambda_1, \lambda_2, \lambda_3) = (23.14, -41.83, 0.09)$.

In the presence of $I_{K2}(V^{(i)})$ ($k^{(i)} \neq 0$, Fig. 1b) that varies strongly with the membrane potential in the vicinity of the equilibrium point however, the unstable node is stabilized without affecting the global flow of the model (Fig. 1d, corresponding eigenvalues $(\lambda_1, \lambda_2, \lambda_3) = (-37.58, -13.6, -0.24)$, estimated for $g_{K2} = 0.2$ at the equilibrium point $(V_0^{(1)}, n_0^{(1)}, S_0^{(1)}) = (-49.084, 0.0027105, 0.19648)$ ¹⁹. $I_{K2}(V^{(i)})$ is given in the form:

$$I_{K2}(V^{(i)}) = g_{K2} p_{\infty}(V^{(i)})(V^{(i)} - V_K) \quad (6)$$

where

$$p_{\infty}(V^{(i)}) = [\exp \frac{V^{(i)} - V_p}{\Theta_p} + \exp \frac{V_p^{(i)} - V}{\Theta_p}]^{-1} \quad (7)$$

represents the opening probability of this channel. In contrast to the other potassium channel (Eqs. 3, 5) which opens with probability $n_{\infty} = 1$ when the membrane voltage reaches a threshold value, the opening probability of the modified channel will be equal only to 0.5¹⁹. From physiological point of view, this can be interpreted as an ion channel dysfunction²⁵ due to non-monotonic voltage dependence,

manifested through a stable silent state. Between the stable node and the bursting attractor there is a rejecting current which enables the system to remain in the stable steady state when starting from initial conditions in its vicinity¹⁹. Generally, the modified model ($k^{(i)} \neq 0$) depicts bistability between physiological, bursting β -cell dynamics and a pathological, silent state dynamics (Fig. 1d).

B. Dynamics of two coupled β -cells under physiological conditions

To investigate the global dynamics of a population of beta-cells, we introduced electrical coupling by adding the following gap-junctional coupling term to the equation for $V^{(i)}$ in (Eqs. 1), that describes bi-directional transport of ions between the cells^{7,26}:

$$I_C(V^{(i)}) = \sum_{j \in \Gamma_i} g_{c,V}(V^{(i)} - V^{(j)}) \quad (8)$$

with $g_{c,V}$ being the coupling conductance (coupling strength). In this way, only electrical coupling of the cells (Fig. 2a) is considered. The sum is taken over the whole population, assuming global coupling.

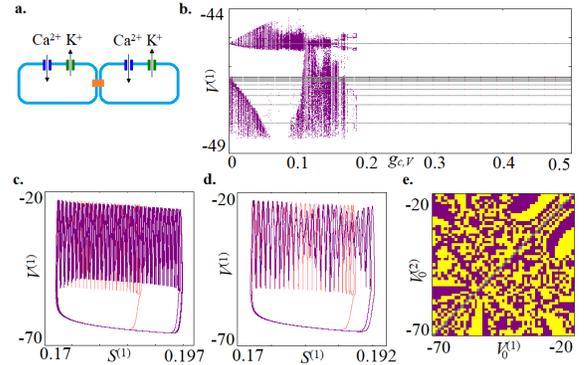


FIG. 2. Behavior of two coupled β -cells with bursting dynamics. **a.** Schematic representation of the model. **b.** Bifurcation trees estimated from Poincaré sections at the hypersurface $n^{(1)} = 0.003$ when $g_{c,V}$ is scanned in both directions (violet: forward, gray: backward). $k^{(i)} = 0, i = 1, 2$ and other parameters as in Fig. 1. Exemplary phase portraits depicting coexistence between synchronized bursting and **c.** periodic or **d.** aperiodic attractors are also shown for $g_{c,V} = 0.03$ and 0.15 respectively. **e.** Estimated basins of attraction of the coexisting bursting attractors for $g_{c,V} = 0.15$ when starting from the following initial conditions: $n_0^{(1)} = 0.003, S_0^{(1)} = 0.1889747, n_0^{(2)} = 0.0025039, S_0^{(2)} = 0.1889565$.

As already noted, if $k_i = 0$ and in absence of coupling, the isolated oscillators display a single stable state - bursting dynamics (Fig. 1c). To infer the effect of coupling on the system's dynamics, we investigated next the bifurcation structure of a minimal model of two coupled cells as a function of

the coupling strength $g_{c,V}$. Bifurcation trees using the membrane voltage variable $V^{(1)}$ were therefore constructed using a Poincaré section at the hypersurface $n^{(1)} = 0.003$ by scanning $g_{c,V} \in [0, 0.6]$ from left to right and vice versa (Fig. 2b violet and grey, respectively). Changing the scanning direction of the bifurcation parameter unravelled that for $g_{c,V} \in [0, 0.19]$, there is a coexistence between a synchronized bursting state and an additional dynamical regime. Numerical simulations for distinct coupling values from this interval showed that this dynamical regime could be periodic or aperiodic (Fig. 2c,d respectively). This is also reflected in the complex fractal structure of the respective basins of attraction, as exemplified for $g_{c,V} = 0.15$ in Fig. 2e. Given that the synchronized bursting state lies only on the diagonal of the basin (grey line in Fig. 2e), it is most probable to reach one of the other dynamical states in this parameter region. For $g_{c,V} > 0.19$ however, only synchronised bursting between both oscillators was observed. This analysis therefore demonstrates that the physiological, synchronized bursting behavior is the dominant dynamical behavior over a wide coupling range in a population of identical β -cells that communicate via gap junctions.

C. Dynamics of two coupled β -cells that contain dysfunctional potassium channels

For $k_i = 1$ however, the presence of the potassium channel which has a decreased opening probability due to non-monotonic voltage dependence (Fig. 1b, Eqs. 6,7) ensures that the equilibrium point is also stabilized, such that isolated oscillators display bistability between a bursting and a silent state in absence of coupling (Fig. 1d). Thus, for initial conditions in the vicinity of the stable node, the physiological bursting behavior is not observed. We therefore refer to this silent regime as pathological. To unravel how such channel defects affect the global dynamics of the population, we used again a minimal model of two coupled β -cells that are individually characterized with bistability between the two states (Fig. 3a), and investigated the two possible scenarios: *i*) the initial conditions of both oscillators are poised either in the bursting attractor or in the stable node attractor, and *ii*) a heterogeneous case where one of the oscillators is poised in the stable node, whereas the other oscillator is poised in the bursting attractor.

When the initial conditions of both oscillators are poised in the same attractor, the solution of the coupled system is trivial - the population behavior is synchronized and identical to the dynamics of the cells in isolation (results not shown). In contrast, for the heterogeneous scenario *ii*), where both cells have same bifurcation structure but populate distinct attractors such that the initial conditions for one cell are poised in the bursting whereas for the other in the stable node, the bifurcation tree constructed analogously to the one in Fig. 2b has a complex structure for varying coupling strength $g_{c,V}$ (Fig. 3b, top). To identify the existing dynamical solutions in this case, we complemented the bifurcation diagram with direct calculations from semi-random initial conditions (Fig. 3b, bottom). In particular, for each $g_{c,V}$ we calculated 500 time series for

the system of two coupled β -cells such that the initial conditions for one of the oscillators were chosen exactly at the stable node $(V_0^{(1)}, n_0^{(1)}, S_0^{(1)}) = (-49.084, 0.0027105, 0.19648)$, whereas the initial conditions for the second oscillator were randomly chosen from a phase space cube in the interval $V_0^{(2)} \in (-60, -20)$, $n_0^{(2)} \in (0, 0.12)$ and $S_0^{(2)} \in (0.17, 0.2)$. Although the size of this cube includes both, the bursting as well as the stable node attractor in the uncoupled case, the basin of the bursting one is dominant. The full set of initial conditions covers the 6-dimensional phase space of the system with 3 degrees of freedom per oscillator densely enough such that one can detect all stable coexisting attractors with a significant basin of attraction. These direct calculations represent a different approach from the bifurcation analysis, and are valid for large system sizes as well²⁷. Therefore, the combination of both methods sheds light on the dynamics of the coupled heterogeneous β -cells model under the introduced pathological channel defects.

The bifurcation tree and the direct calculations plot (Fig. 3b) demonstrate that under weak coupling ($g_{c,V} < 0.017$), the heterogeneous system of two coupled β -cells where initial conditions differentially poise the oscillators in the two stable attractors, predominantly displays either bursting or complex, aperiodic dynamic. For the $g_{c,V}$ sub-intervals for which bursting or aperiodic dynamics was observed, the probability for the coupled system to exhibit steady state solution was less than 10%. That the bursting or the aperiodic dynamics are the dominant regimes under weak coupling is also reflected in the corresponding basins of attraction estimated for the respective $g_{c,V}$ values (Fig. 3c,d). In contrast, for a large part of intermediate coupling strength interval ($0.017 < g_{c,V} < 0.08$), the probability that the coupled system exhibits steady state dynamics was $\sim 100\%$, with less than 10% variability in small $g_{c,V}$ ranges. The exemplary basin of attraction plot estimated for $g_{c,V} = 0.02$ also depicts the dominance of the stable equilibrium point, with only small lines of the basin of the bursting attractor (Fig. 3e). This is surprising, given the small basin of attraction of the steady state for isolated cells (Fig. 1d). The mechanism leading to this stabilization will be discussed elsewhere (manuscript in preparation).

A further increase in $g_{c,V}$ continuously decreases the probability for the full system to exhibit steady state behavior. At $g_{c,V} \approx 0.12$ for example, the system visits the bursting attractor and the equilibrium point with equal probability. This implies that in the presence of noise, the full system can switch between bursting physiological and silent pathological state.

For high enough coupling strength $g_{c,V}$ however, the basin of attraction of the steady state is significantly decreased, leaving the bursting attractor as the most common solution of the system (Fig. 3f). These results therefore demonstrate that a heterogeneous system of two coupled β -cells that have dysfunctional potassium channels and thereby possibility to manifest a stable steady state, predominantly exhibits physiological bursting dynamics. This is true for weak and strong coupling strengths, even when the initial conditions of one of the oscillators correspond to its positioning in the stable node. Only for intermediate coupling values the silent pathological state is the dominant dynamical regime, even for small steady

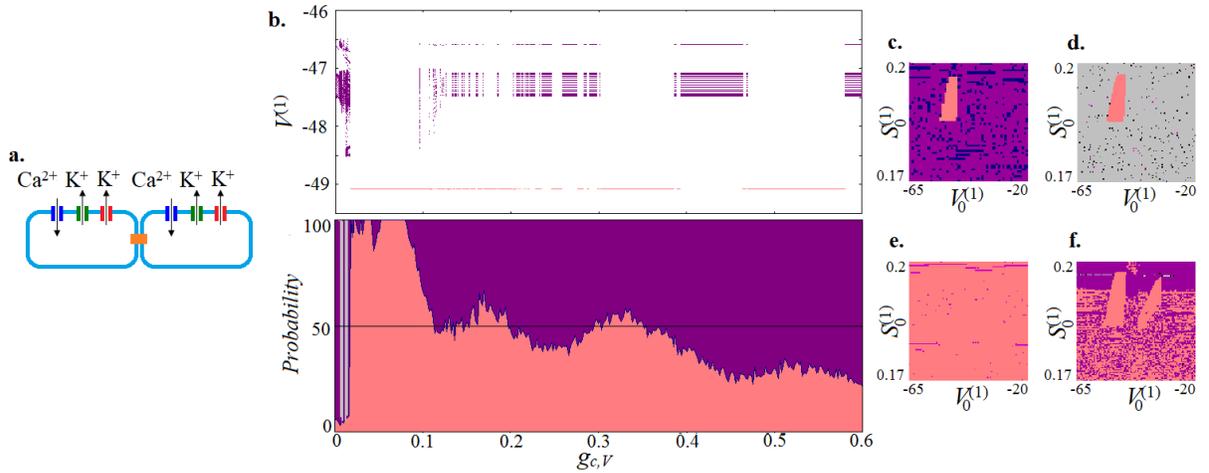


FIG. 3. Behavior of two coupled β -cells with dysfunctional potassium channels. **a.** Schematic representation of the two-cell system. **b.** (Top) Bifurcation tree estimated from Poincaré sections at the hypersurface and $n^{(1)} = 0.003$ constructed for fixed initial conditions: $V_0^{(1)} = -50.784$, $n_0^{(1)} = 0.0245$, $S_0^{(1)} = 0.1774$, $V_0^{(2)} = -49.084$, $n_0^{(2)} = 0.027105$, $S_0^{(2)} = 0.19648$, as well as (bottom) probability of appearing of co-existing attractors in dependence on the coupling strength $g_{c,V}$. $k^{(i)} = 1, i = 1, 2$ and other parameters as in Fig. 1. Exemplary basins of coexisting attractors for various strength of coupling: **c.** $g_{c,V} = 0.004$; **d.** $g_{c,V} = 0.00663$; **e.** $g_{c,V} = 0.02$; **f.** $g_{c,V} = 0.12$. For **c.-f.**, the initial conditions of the other variables were fixed in steady state $n_0^{(1)} = 0.027105$, $V_0^{(2)} = -49.084$, $n_0^{(2)} = 0.027105$, $S_0^{(2)} = 0.19648$. Grey: aperiodic dynamics; Violet: bursting dynamics; Red: steady state.

state basin of attraction for single cells.

D. Dynamical behavior of a minimal mixed population model

To model physiologically more realistic scenario, we investigate next the dynamics of a minimal mixed system of two coupled cells, where only one of them displays bistability between bursting and a silent state, and thus a channel function dysregulation (Fig. 4a). Thus, in contrast to the previous scenario, the coupled cells have a different bifurcation structure.

Bifurcation analysis showed that the mixed population does not display steady state dynamics for any $g_{c,V}$ value in the given interval (Fig. 4b). This is contrary to the previous case (Fig. 3b) where silent state was dominant for intermediate $g_{c,V}$ values when bistability characterizes both cells, although the initial conditions of only one of them were poised in the stable node (coupled heterogeneous β -cells). A typical dynamics of the coupled mixed population model is mixed-mode oscillations, defined as complex oscillatory patterns where small-amplitude high-frequency oscillations alternate with large-amplitude long-period cycles²⁸. Increasing the coupling strength between the two mixed oscillators induces transitions between different manifestations of the mixed-mode oscillations. For example, under low coupling strength, one oscillator displays small-amplitude whereas the other large-amplitude bursting dynamics (Fig. 4c). For intermediate coupling, different patterns of alternating between the small- and the large-amplitude bursting occur (Fig. 4d-f), to finally converge to almost synchronized bursting of both oscillators under strong coupling (Fig. 4g). The analysis therefore demonstrates that in a mixed population of two coupled

cells, where only one of them has dysregulation of a potassium channel, pathological silent dynamics is not stabilized on the level of the population. This implies that deviations from the physiological bursting dynamics might be possible for a larger sub-population size that has channels dysregulation.

E. Effect of single cell channel dysregulation on the dynamics of mixed β -cells population

To identify whether stabilizing pathological dynamics is a system-size effect, we investigated the dynamics of mixed populations of increasing sizes, starting with a minimal population of $N = 3$ cells, where at least $(N/2 + 1)$ of them can exhibit bistability between silent and bursting dynamics. For simplicity, we initially assume global coupling between all cells. The numerical simulations demonstrated that in this case there is a threshold of coupling strength for which the silent state can be stabilized on the population level (Fig. 5a). However, the probability to observe this pathological, silent dynamics in the mixed cell population is significantly smaller than the one in the heterogeneous population (Fig. 3b). Direct calculations from semi-random initial conditions as in Fig. 3b showed that for the minimal system of $N = 3$ cells, this probability is smaller than 1% (Fig. 5b). Subsequent increase of the population size by a cell that can display bistability between a silent and bursting dynamics in turn lowered the coupling threshold for which the pathological, silent state was observed (Fig. 5a, bottom). The simulations also demonstrated that the probability to observe the pathological state increased with the increase of fraction of cells that have the channel defects. For

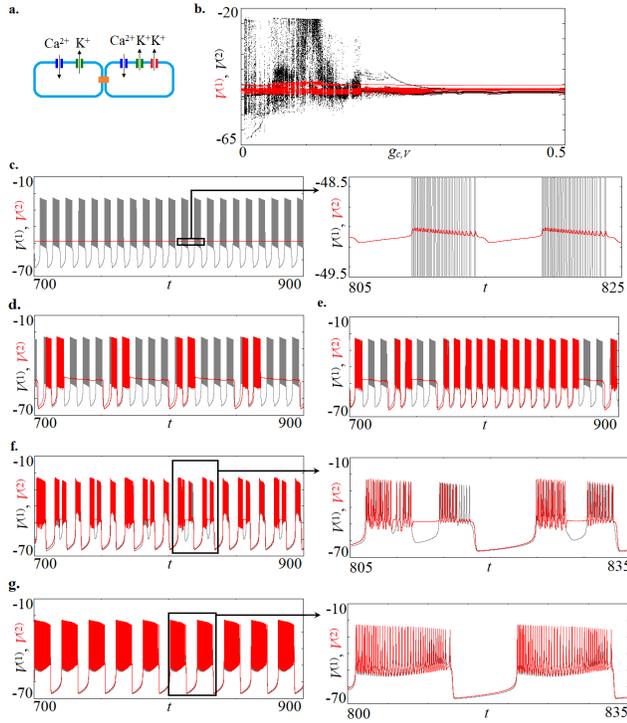


FIG. 4. Behavior of a minimal mixed population model. **a.** Schematic representation of minimal mixed population of two coupled cells (only one cell has channel dysregulation). **b.** Bifurcation tree estimated from Poincaré sections at the hypersurface $n^{(1)} = 0.003$ when $g_{c,v}$ is scanned in both directions. $k^{(1)} = 0, k^{(2)} = 1$ and other parameters as in Fig. 1. Exemplary time series of characteristic dynamical regimes for: **c.** $g_{c,v} = 0.001$ **d.** $g_{c,v} = 0.008$; **e.** $g_{c,v} = 0.015$; **f.** $g_{c,v} = 0.07$; **g.** $g_{c,v} = 0.3$.

example, for a mixed population of $N = 12$ cells, where 11 of them have dysfunctional channel, the probability of observe the silent dynamics increased to approximately 10%, almost over the full range of coupling strengths (Fig. 5c).

Physiological studies of β -cells activity in the islets of Langerhans have however shown that injection of a dye through a microelectrode in a single cell usually labels a small number of neighboring cells^{29,30}, presumably by passing through the gap junctions. It has been therefore suggested that direct local coupling gives rise to the emergent dynamical behavior of β -cells¹⁷. We have therefore repeated the above simulations when nearest-neighbor coupling was considered, where each cell is coupled with 6 neighboring cells. Restricting the coupling to local interactions did not result in qualitatively different behavior than the one observed for global coupling. In this case, for example for a mixed population of $N = 12$ cells where 11 have a dysfunctional channel, the probability to observe the silent dynamics is slightly smaller than 10% (Fig. 5d), which was observed for the globally coupled case. Thus, despite the increase in the probability to observe the pathological state with increased system size, the population of mixed cells, being globally or locally coupled, can still cooperatively regulate the physiological bursting dynamics, even when the majority of the population has acquired

dysregulation in the channel's activity. This is contrary to the dynamics of the population where each cells has dysregulation, in which case a coupling interval exists for which the probability to observe the pathological dynamics is $\sim 100\%$ (Fig. 3b).

When the number of cells that lack dysfunctional channels in the heterogeneous population is however increased, we find that the coupling threshold for which stabilization of the silent state is reached is subsequently increased.

Indeed, in accordance with the previous finding that at least $(N/2 + 1)$ cells must have dysfunctional channels for the silent state to be stabilized, the numerical simulations for globally coupled systems demonstrated that when 2 or 3 cells display only bursting dynamics, the minimal mixed population size for which the silent state can be stabilized is $N = 5$ (Fig. 6a,c) or 7 (Fig. 6b,d) cells, respectively. The stabilization threshold scales with the number of cells displaying only bursting dynamics, being highest when the mixed population has the largest number of cells exhibiting solely bursting dynamics. Similarly to Fig. 5a, a step-wise increase of the population by a cell that displays bistability between the silent and the bursting state also decreases the coupling threshold for stabilization (Fig. 6c,d). The local coupling did not affect the qualitative behavior of the mixed population in this case as well (Fig. 6c,d).

III. DISCUSSION

Intercellular communication dictates coordinated dynamics of cellular populations, and thereby determines cellular identity in terms of their function. For example, regulated insulin secretion of pancreatic β -cells is driven by well-synchronized bursting activity of the population, where the secretion increases with the fraction of time the cells spend in the spiking state. In turn, disassociated β -cells also show increased electrical activity at elevated glucose concentrations, however the activity varies from cell to cell, such that some cells are electrically “silent” whereas others are continuously “spiking” at any given glucose concentration^{10–12}. Using transgenic mice where the potassium channels are functionally “knocked out” in 70% of the β -cells, it has been however demonstrated that the intact islets still show glucose-dependent insulin secretion¹⁵, suggesting that the functional identity of β -cells can be dynamically maintained on a population level due to cell-cell coupling.

Here, we investigated the question how channel defects on the level of single cells that alter the intercellular communication can affect the global dynamics of the population and under which conditions the dynamics will deviate from the physiological one. An altered dynamics would in turn affect the cellular function and thereby their identity. The bifurcation analysis of a minimal homogeneous or heterogeneous populations of $N = 2$ coupled cells, where respectively either both cells have channel defects and thereby can exhibit silent state or only one of them, demonstrated very different dynamical behavior on the population level. Whereas the homogeneous population exhibited dominance of the patholog-

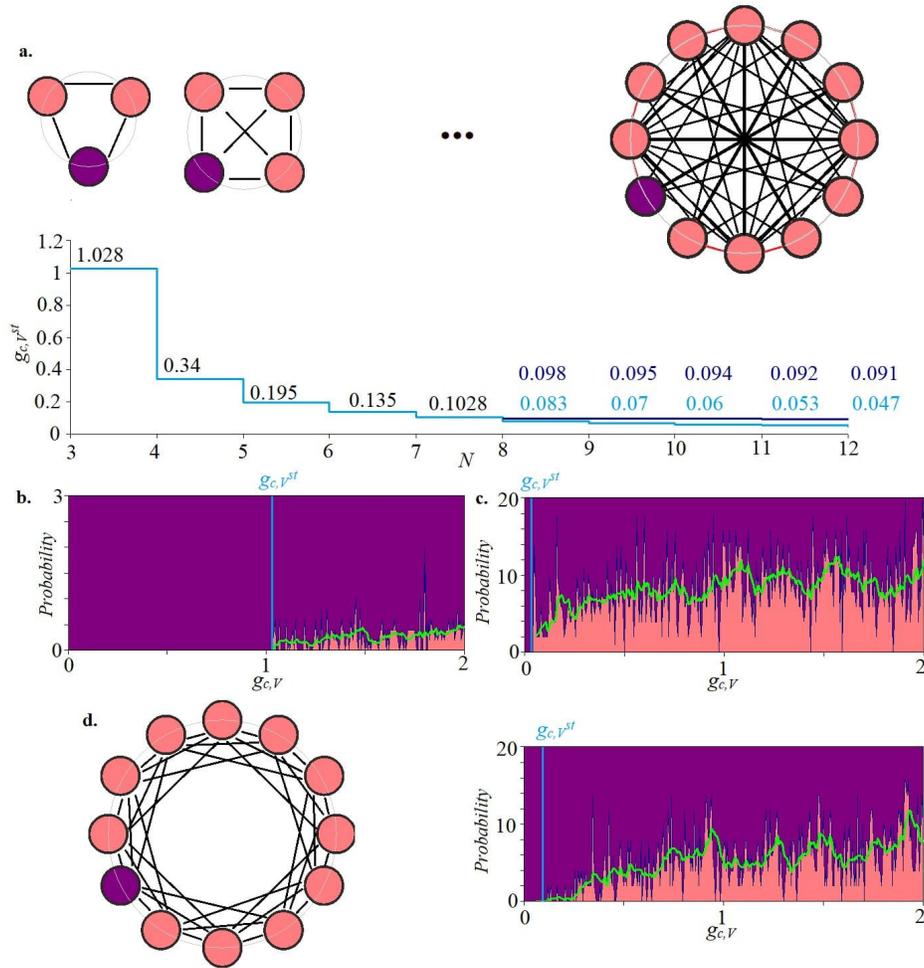


FIG. 5. Behavior of mixed β -cells population. **a.** (Top) Schematic representation of globally coupled networks of increasing size, where only a single cell does not have a channel dysregulations (red color of node responds to $k^{(i)} = 1$, violet color to $k^{(i)} = 0$) and (bottom) a corresponding plots of coupling strength thresholds for steady state stabilization in dependence on the size of the population (N) for globally (light blue) and locally (dark blue) coupled cells. Schematic representation of locally coupled population in Fig. 5d, left. Probability of appearing of co-existing attractors in dependence on the coupling strength for the minimal population **b.** of $N = 3$ coupled cells, as well as for **c.** $N = 12$ globally or **d.** locally coupled cells. Other parameters as in Fig. 1. $g_{c,V}^{st} - g_{c,V}$ threshold for which the steady state is stabilized.

ical silent state ($\sim 100\%$) for a given coupling range despite the small basin of attraction on a single cell level, the minimal heterogeneous population was characterized only with bursting dynamics over the full coupling range. These findings resemble previously observed cooperative effects in heterogeneous populations of coupled β -cells. In particular, it has been demonstrated that bursting on a population level can be restored due to cell-cell coupling, although single cells can only spike continuously or be silent^{9,31}. Similar effects have been also observed for coupled van der Pol-FitzHugh-Nagumo units³². Although in these cases the nature of the bistability between the limit cycle and the steady state was different than in the model used here, what is equivalent is that all of the cells had an equivalent bifurcation structure, where the heterogeneity was introduced by the parametric positioning of the system in one of the stable attractors.

Our findings therefore suggested that the pathological state

can only be stabilized as a system-size effect in a mixed population of cells. In particular, we coupled cells with different bifurcation structures: cells that only exhibit bursting, and cells that have bistability between bursting and silent state, and investigated the effect of the size of the subpopulation which has channel defects on the cooperative dynamics of the full system. For a mixed population of two cells, a coupling range was identified where the silent mode of operation was dominant. The numerical simulations of larger mixed populations on the other hand demonstrated that although the silent state could be indeed stabilized when the fraction of cells with channel dysfunctions is increased, the bursting dynamics still remained a dominant dynamical behavior of the population, with prevalence of $\sim 80 - 90\%$. Thus, the physiological population dynamics and the thereby associated cellular function, such as insulin production in the case of bursting dynamics of β -cells can be reliably maintained, even in the presence of

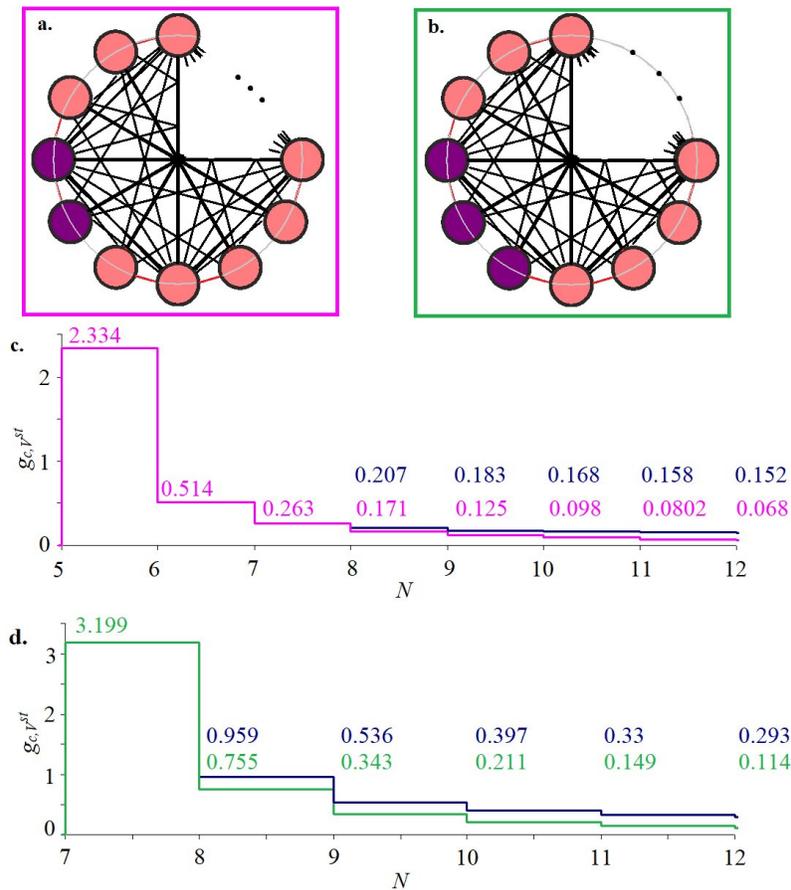


FIG. 6. Behavior of mixed β -cells population as depending on the number of cells without channel dysregulations. **a,b.** Schematic representation of networks of interacting cells (red color for $k^{(i)} = 1$, violet color for $k^{(i)} = 0$) with different number of cells displaying only bursting dynamics. Corresponding plots depicting the thresholds of coupling strength for which the steady state is stabilized as a function of the size of the population (N) when **c.** $N = 2$ or **d.** $N = 3$ cells display only bursting dynamics. In **c-d**, pink and green lines correspond to global, whereas dark blue - local coupling. Other parameters as in Fig. 1.

dysfunctional channels on the level of single cells.

ACKNOWLEDGMENTS

N.S thanks the DAAD (Project Nr. 57440915) for the financial support of the scientific visit to MPI-MP in Dortmund, 2019.

AUTHOR CONTRIBUTIONS

A.K. conceptualized the study. N.S. performed the numerical simulations and bifurcation analysis with help of A.K. N.S. and A.K. interpreted the results and wrote the manuscript.

CONFLICT OF INTEREST

The authors declare that they have no conflict of interest.

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