The simplest oscillators with adaptive properties

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***Abstract*— We study the behavior of a non-autonomous oscillator with phase of the external force having the property of adaptability, i.e. depends on the dynamic variable. In the frame of this work we consider the case when dependence of the phase of external force is polynomial including terms of the third degree. Such dependence of the phase of the external action leads to the appearance of the complex chaotic oscillations in the dynamics of the oscillator. In the parameter space a hierarchy of various periodic and chaotic oscillations is observed. It is shown that in the dynamics of the system oscillation modes are observed, similar to the modes of a non-autonomous oscillator with a potential in the form of a periodic function. In the case of a linear adaptation function, the structure of the parameter plane has characteristic cascades of period doubling bifurcations. When nonlinearities (the second and the third terms of polynomial function) are taken into account, structures are destroyed.**

Keywords— adaptive property, non-autonomous oscillator, chaos

I. Introduction

Adaptive properties of dynamical systems are a change in the behavior of a system in response to a change in its dynamical variables. Different systems in biology, physics, chemestry, and others, exhibit oscillatory behavior in which one the system acts on another with an external signal, but when the operating conditions change, the frequency of forcing adjustes. As such example we can recall so-called phase-locked loop, which is used in information transmission systems to ensure its high stability [1-2]. Another example is the system of cardiovascular regulation of living organisms, when changing in load increases or decreases the heart rate [3-5]. Adjustment is a general principle in the nervous system and occurs at different levels of information processing and in a wide range of different time scales. This kind of adjustment can be called adaptation of the system to external stimuli, while the system itself has adaptive properties [6-8]. Adaptation is not a clearly defined term. Nonetheless, there are examples that are considered adaptive. For example, the evolution of species, a change in the state of a neuron during stimulation, and many others can be considered as adaptive behavior. In all cases, adaptation can be understood as a process of optimization or adaptation. In evolution, the phenotype of a species changes to optimize survival in a changing environment.

Another example of adaptive behavior is the movement of a particle in a potential field. The simplest adaptive dynamical system consists of a one-dimensional potential that completely determines the motion of the particle, and a dynamic rule as the potential changes over time. One option is a sinusoidal potential, which changes slowly depending on the position of the particle. The simplest example is forced oscillations of harmonic oscillator.

The dependence of the phase of external force on the dynamic variable can lead to the occurrence of chaotic behavior in the system. The control in this case is very complicated, its study and modeling confront a number of difficulties. One of the ways in the investigation of such systems and processes is the examination of simpler objects in which the self-sustained oscillations and frequency control of external force are rather easily modeled. The convenient paradigmatic model is the classical model of the theory of oscillations and waves - a linear dissipative oscillator under external harmonic force.

II. Object of Investigation

A. Mathematicla model of generator

As starting point let us consider the equation of a non-autonomous linear oscillator [2], excited by an external harmonic signal in the next form:

$$\ddot{x} + 2\alpha\dot{x} + \omega_0^2 x = A\sin(\omega t + \varphi)\,. \tag{1}$$

Here t, x, $\dot{x}$ are dynamic variables of oscillator, ω_0 is the natural frequency of the oscillations of the circuit, α is the coefficient of dissipation, A, ω and φ are the amplitude, frequency, and phase of the external force, accordingly. Let assume that the phase of external force depend on the dynamic variable. Then equation (1) we can rewrite in a new form:

$$\ddot{x} + 2\alpha\dot{x} + \omega_0^2 x = A\sin(\omega t + \varphi(x))\,. \tag{2}$$

In the general case, the dependence of the phase on the dynamic variable can be complex, and traditionally this dependence has polynomial form:

$$\varphi(x) = k_1 x + k_2 x^2 + k_3 x^3\,, \tag{3}$$

where k_1, k_2, k_3 are constant coefficients, $x(t)$ is the dynamic variable of equation (1). After passing to dimensionless time $\tau = \omega_0 t$, equation (1) takes the form

$$\ddot{x} + 2\alpha\dot{x} + x = A\sin(p\tau + k_1 x + k_2 x^2 + k_3 x^3)\,, \tag{4}$$

where $p=\omega/\omega_0$ is the normalized frequency of external force. Equation (4) has a nonlinearity sin(kx), thus, our linear non-autonomous oscillator became nonlinear non-autonomous. For $k_2 = 0$, $k_3 = 0$, the phase dependence of the variable will be linear. It was shown in [9-10] that even with a linear dependence in the system, the appearance of complex periodic and chaotic oscillations is possible.

III. Numerical Simultions

A. Linear function

Firstly let us consider features for the case when dependence of phase on dynamical variable is linear, $k_1=k$, $k_2=0$, $k_3=0$. In Fig. 1, on the parameter plane (p, A), chart of dynamical modes are presented in the case of a linear dependence of the phase of the action on the dynamic variable of equation (4). Different colors indicate the areas of periodic oscillation (detailed description see in palette in Fig.1), chaotic oscillations are marked by gray color (C). The main scenario of the transition to chaos in the system is a cascade of period-doubling bifurcations. In the plane, the main domains of complex behavior is associated with the so-called resonances at higher harmonics. The transition between these areas is hysteretic in nature. It should be noted that similar transitions exist inside each zone, as a result of bi- and multistability in the system under study. A detailed analysis of the structure of each of the zones of complex modes of oscillations shows that the dynamics of the system exhibits a complex hierarchy of regions of periodic and chaotic oscillations. In general, the structure of the plane of control parameters in many respects resembles that for a non-autonomous nonlinear oscillator [11-12].

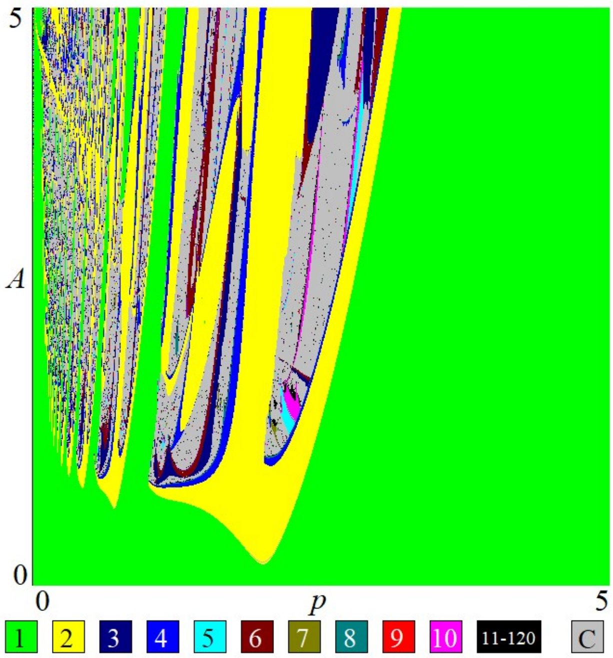

Fig. 1. Chart of dynamical modes for adaptive non-autonomous harmonic oscillator (4), α=0.1, k_1=1, k_2=0, k_3=0.

B. Polinomial function

Now let us take into account nonlinear character of the dependence of phase on one of the dynamical variables (x). Firstly we consider the case when linear component is persist, and we take into account quadratic and cubic terms of polynom. In Fig.2 two chart of dynamical modes are presented for case when linear and quadratic components are non-zero (Fig.2a), when linear and cubic components are considered (Fig.2b), and all three components involved in dynamics. In the case of a nonlinear dependence, the parameter plane structure is complicated. Thus, introducing of quadratic and cubic nonlinearity leads to a change in the configuration of the domains of periodic and chaotic oscillations in the parameter plane. A significant extension of the domains of chaotic oscillations and a compression of the regions of periodic oscillations are noted. With the introduction of a nonlinear dependence of the phase on the dynamic variable, the shape of the oscillations and the type of the phase portrait practically do not change, however, some expansion of the spectrum of chaotic oscillations should be noted. On the whole, the dynamics of the system do not change qualitatively, the transition to chaos occurs as a result of bifurcations of doubling the oscillation period, bi and multistability take place in the space of control parameters.

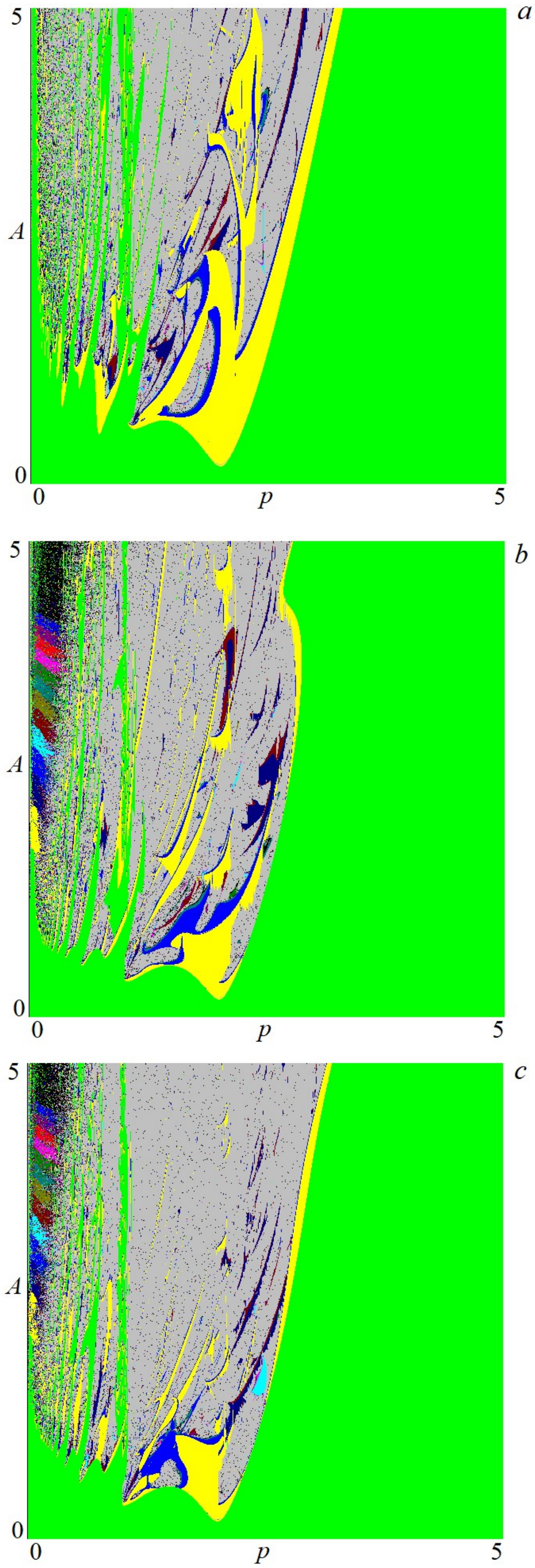

Fig. 2. dynamical modes for adaptive non-autonomous harmonic oscillator (4), α=0.1, a) k_1=1, k_2=1, k_3=0; b) k_1=1, k_2=0, k_3=1; c) k_1=1, k_2=1, k_3=1.

Then we consider the case when linear component is excluded. In Fig.3 charts of dynamical modes for this cases are shown. Fig.3a shows the structure when dependence of phase on dynamical variable is quadratic. In this case we did not observed lines of period-doubling bifurcations. It means that cascade of period doubling goes with very small variation of parameters, or transition from periodic oscilla-

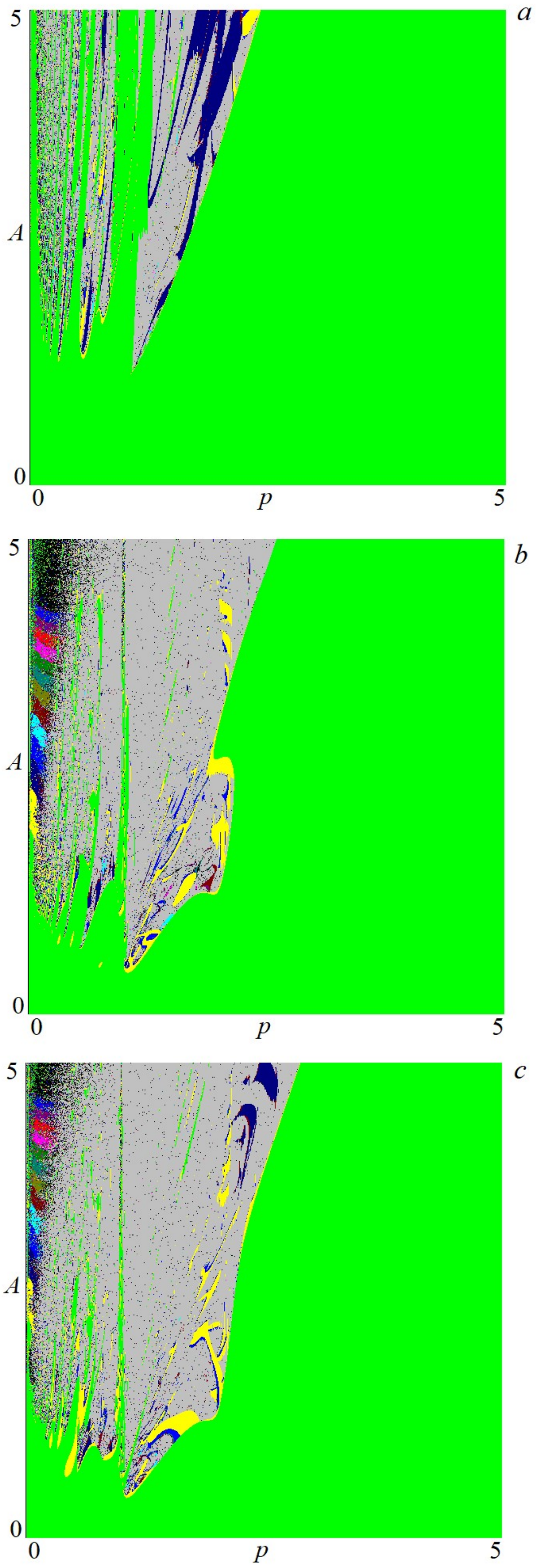

Fig. 3. dynamical modes for adaptive non-autonomous harmonic oscillator (4), α=0.1, a) k_1=0, k_2=1, k_3=0; b) k_1=0, k_2=0, k_3=1; c) k_1=0, k_2=1, k_3=1.

tions to chaotic is hard. This point will be clarified in the talk. Fig.3b demonstrates the chart of dynamical modes for the case when adaptive function is cubic. In this case on the boundary of domain of periodic oscillation one can see lines of period-doubling bifurcations, but it is also destroyed for more high frequency of external force. In Fig. 3c the chart of dynamical modes for case when adaptive function include second and third terms of polynom. In this case we can see that the most lines of period-doubling bifurcations are destroyed, there are a few lines which are persist.

Thus, the introducing of the dependence of the external action phase on the dynamic variable in a non-autonomous linear oscillator leads to its transformation into a non-linear non-autonomous oscillator with sin(kx) nonlinearity. In the parameter space a hierarchy of periodic and chaotic oscillations is formed. In the case of a linear dependence of the phase on the one of the dynamic variables (x), the structure of the parameter space qualitatively corresponds to that for a non-autonomous nonlinear oscillator (like Duffing, or van der Pol). With the taking into account a nonlinear dependence of the phase on the dynamic variable, a change in the configuration of the regions of existence of complex oscillations modes is observed, as well as an expansion of the domains of chaotic oscillations.

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