Routes to chaos in the nonholonomic model of Chaplygin top

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We consider the motion of a dynamically asymmetrical ball on a plane in the gravity field. The center of mass of the ball does not lie on any planes of inertia, and the point of contact of the ball with the plane is subject to a nonholonomic constraint which forbids slipping. Following [1] we call such a ball Chaplygin top.

The aim of this study is to investigate the typical scenarios of the appearance and evolution of strange attractors in the nonholonomic model of Chaplygin top. Our interest in nonholonomic models is caused by the fact that (as was shown in previous studies [2, 3]) such systems exhibit a wide variety of new interesting examples of strange attractors that are typical for the three-dimensional maps [4]. For example, our recent research [5] shows that the nonholonomic model of Chaplygin top demonstrates the so called "figure-eight" strange attractor, which relates to pseudohyperbolic strange attractors [4].

Here we show that the nonholonomic model of Chaplygin top demonstrates a comprehensive variety of scenarios of torus attractors breakup, in particular, in accordance with the mechanism of Afraimovich-Shilnikov [6], including Feigenbaum cascade inside the synchronization domain, and via torus doubling cascade [7]. In addition, the model exhibits some typical sequences of bifurcations of regular and chaotic attractors, which include the above basic scenarios of tori destruction as their stages. One of such metascenarios results in a discrete heteroclinic Shilnikov attractor [4], Fig. 1.

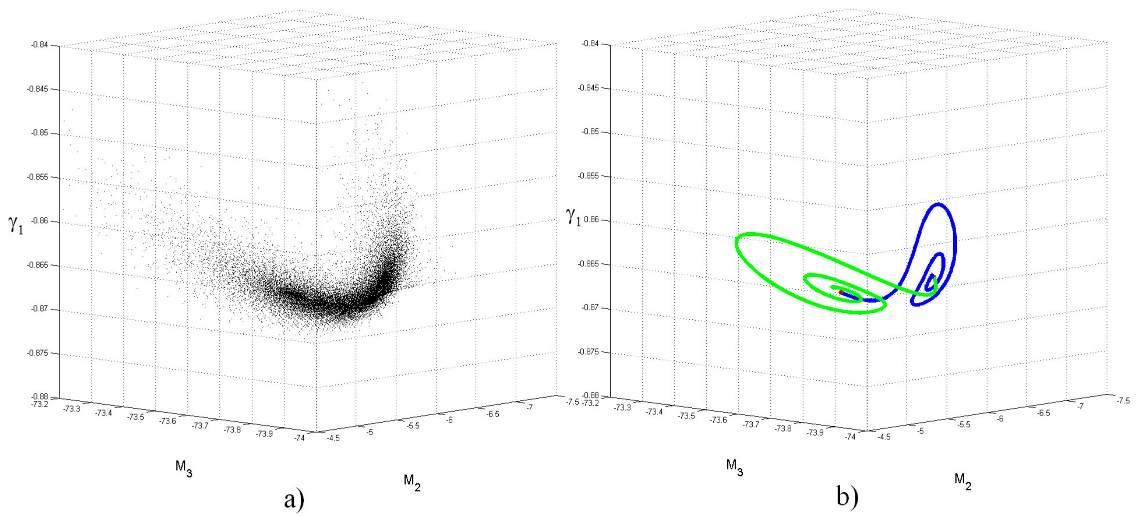


Fig. 1. a) Discrete heteroclinic Shilnikov attractor; b) heteroclinic cycle.

Another feature of the dynamics of nonholonomic model of Chaplygin top is the presence of a developed multistability. Evolution of coexisting attractors may here proceed in accordance with the scenario, which results in a strange attractor, that coincides with the homo-(hetero-)clinical structure of saddle limit cycle, which lies on the border of the basins of attraction of initially coexisting attractors. One such scenario was found to occur in the model under investigation. It results in the chaotic ring heteroclinic attractor, Fig. 2.

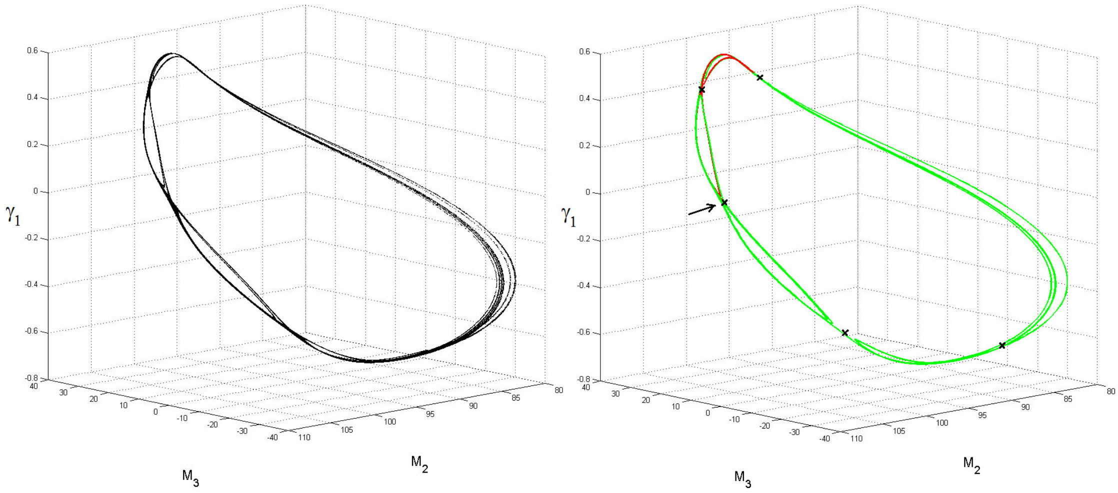


Fig. 2. a) Ring heteroclinic attractor; b) unstable invariant manifolds of the saddle cycle.

This work was supported by RSF grant No 15-12-20035.

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