Nonautonomous dynamics of coupled Van der Pol oscillators in the regime of amplitude death.

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Abstract

Pulsed driven system of two coupled Van der Pol oscillators in the regime of amplitude death is researched. The existence of islands of quasiperiodic regimes on the parameter plane period amplitude of external action in the radiophysics experiment are shown. The different types of oscillations in this system are illustrated.

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1 INTRODUCTION

Recently, the study of coupled nonlinear dynamical systems has gained extensive attention. Natural systems are rarely isolated, and thus studies of coupled dynamical systems of different nature (physical, chemical, social, etc.) are important for many areas of science [1, 2]. Interaction of coupled self-oscillatory systems leads to new phenomena such as synchronization, hysteresis, phase multistability, etc. So, one of the important phenomenon, which can occur in coupled self-oscillatory systems is amplitude death (oscillator death) [3]-[10]. Amplitude death can occur when interaction of oscillators causes a fixed point to become stable and attracting. First observed of this effect was experimentally in chemical systems [3]. Now, the effect of amplitude death is studied from various aspects in many papers [3]-[10].

The simplest case when in the system can occur the effect of amplitude death is dissipatively coupled self-sustained oscillators with different base frequencies. The coupling between oscillators has to be enough strong, as it introduces in each oscillator additional damping, which cant to be compensated for energy of another oscillator. A theoretical analysis of this effect was given in [1, 4]. Also it was observed in the series of physical systems, for example, in a coupled thermo-optical oscillators [5].

In [6] showed that amplitude death can occur in identical systems if the coupling is nonlinear or the interaction is delayed. For coupling systems with delay this effect can occur not only when the autonomous subsystems demonstrate periodic behavior, but also for oscillators in chaotic regime. The effect of amplitude death observed for chaotic Rossler and Lorenz systems [7]. Note that this effect observed for coupling multi-type systems, for example, Van der Pol oscillator and brusselator or Van der Pol oscillator and generator of Kislov-Dmitriev [8].

Thus, the effect of amplitude death is common for systems of coupled self-sustained oscillators. It prove the fact that this effect was revealed in the experimental works for systems of different natures: chemical [4], physical [5], electro-chemical and electro-biological [9].

The question about dynamics of coupled oscillators in the regime of amplitude death driven external force is enough interesting. It seems that the system in the regime of amplitude death under periodic action will be similar a nonautonomous dissipative oscillator. It turns out that it is not. In [11] was showed, that adding external force can lead not only simple regular oscillations, but it can initialize quasiperiodic regimes. In the present work we characterize in detail this regimes. In Section 1 we produce the results of numerical simulation for system of two couple Van der Pol oscillators driven periodic force. In Section 2 we represent the results of experimental studying of two coupled Van der Pol oscillators driven periodic pulsed force and comparison of numerical and experimental results.

2 NUMERICAL SIMULATION

Let us consider the system of two coupled Van der Pol oscillators driven pulsed action:

$$\begin{aligned} &\ddot{x} - (\lambda - x^2)\dot{x} + x + \mu(\dot{x} - \dot{y}) = A \sum \delta(t - nT), \\ &\ddot{y} - (\lambda - y^2)\dot{y} + (1 + \Delta)y + \mu(\dot{y} - \dot{x}) = 0, \end{aligned} \tag{1}$$

Here x, y are dynamical variables of each oscillator, λ is parameter characterizing increase under threshold of Andronov-Hopf bifurcation in the uncoupled oscillators; Δ is frequency detuning of the second oscillator relative of the first oscillator for uncoupled systems; μ is the coefficient of dissipative coupling ; A is amplitude; T is period of external force. External force we choose such short pulses, it can be represented by the sequences of δ-functions.

Let us consider a chart of dynamical regimes on the parameter plane of the period and amplitude of external force (T, A) shown in Fig. 1. This chart constructed in the next way: in each point of parameter plane using stroboscopic section of Poincare we determined period of regime, realizing in the system. In corresponding with it each point of parameter plane colored in different colors. Non-periodic regimes (including quasiperiodicity and chaos) are shown in gray. Other colors are chosen in dependence on the period of oscillations. Light green color designates the regimes of period 1:1.

In Fig. 1a we can see the chart of dynamical regimes for the next parameter of system (1): $\lambda = 1$, $\mu = 1.3$, $\Delta = 4$. In this case in autonomous system realized regime of synchronization 1:1. Fig. 1b correspond situation when in autonomous system realized regime of oscillator death ($\lambda = 1$, $\mu = 1.3$, $\Delta = 6$). We can notice some features of this pictures. When autonomous system demonstrate regime of self-oscillations, the parameter plane (T, A) is similar for case when single Van der Pol oscillator driven pulsed action [12]. There is set of synchronization tongues (main resonances and subharmonic resonances), outside tongues we can observe quasiperiodic regimes.

Transition of coupled oscillators to the area of amplitude death regime (Fig. 1b) is accompanied by occurrence amplitude threshold of initiation of quasiperiodic regimes. In this case area of quasiperiodic regimes has form isolated islands. Subharmonic resonance tongues transform to some narrow periodic windows, which intersect the island of quasiperiodicity. In Fig. 2 stroboscopic section of Poincare for this periodic window, corresponding 2-cycle (In Fig.2 it marks by arrow and Letters P1, P2) and invariant curve

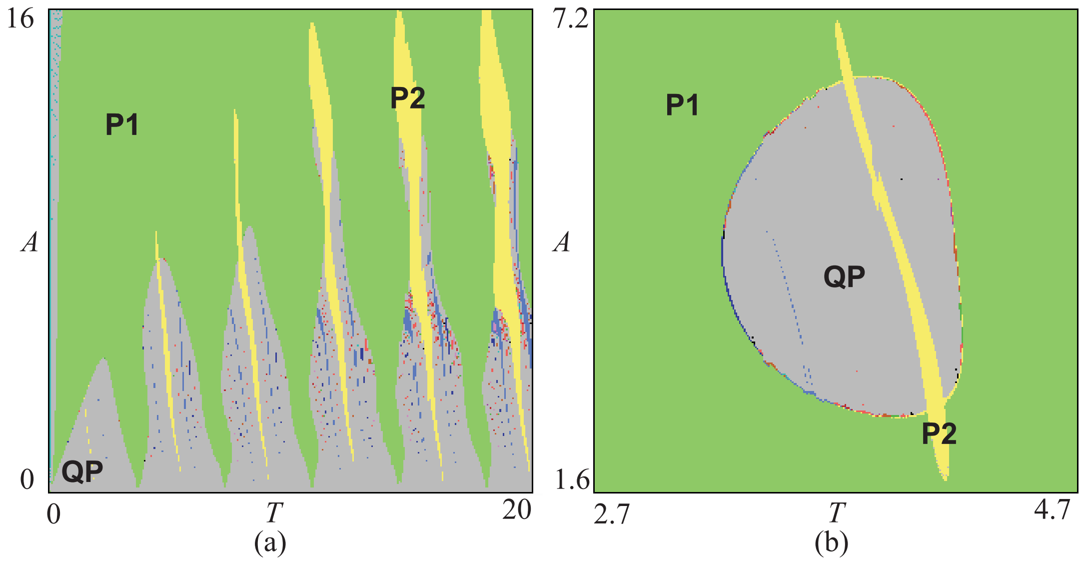

Figure 1: Charts of dynamical regimes for system (1) on the parameter plane period T - amplitude A of external force: a) autonomous system demonstrates regime of synchronization: $\lambda = 1$, $\mu = 1.3$, $\Delta = 4$; b) autonomous system demonstrates regime of amplitude death: $\lambda = 1$, $\mu = 1.3$, $\Delta = 6$.

(T). We can see, that periodic regimes are the result of synchronization in invariant curve (for full system is torus).

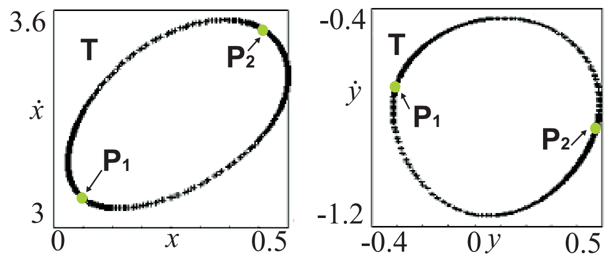

Figure 2: Projections of stroboscopic sections of Poincar on planes $(x,\, \dot{x})$ and $(y,\, \dot{y})$. T invariant curve for quasiperiodic regime; P1, P2 fixed points of 2-cycle.

Let us turn to analyze of Lyapunov exponents. The graphics of dependence of two largest Lyapunov exponents on period of external force is given in Fig.3. Since the system (1) is nonautonomous, the dimension of its phase space $N = 5$, correspondingly, it has five Lyapunov exponents, one of them equal zero always, and two smallest exponents are negative always. Dependence of two residual largest exponents on period of external force determinates the regime realizing in the system. In Fig.3 first (the largest) Lyapunov exponent colored in gray, second Lyapunov exponent colored in black; the area when first and second exponents are equal marks by the points. In Fig.3 we can see that both of Lyapunov exponents are negative always, but the firs become zero in some interval of parameter T. This fact proves that observed regime is quasiperiodic. Also, we can see the interval where both Lyapunov exponents are negative, this interval correspond tongue of synchronization of period-2.

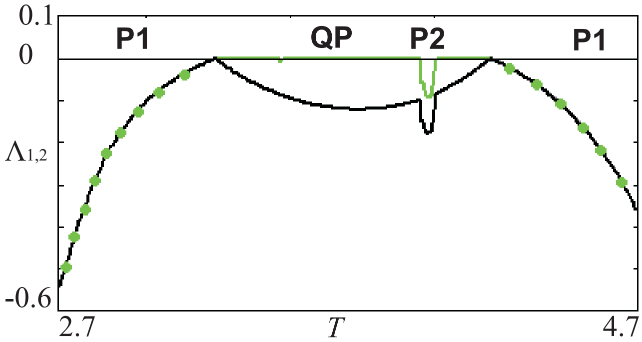

Figure 3: Dependence of two largest Lyapunov exponents on period of external force T at line of constant amplitude $A = 4.3$; $\lambda = 1$, $\mu = 1.3$, $\Delta = 6$.

At further advance to the area of amplitude death in the system (1) without external force, i.e. increasing of frequency detuning of oscillators , in nonautonomous system (1) the islands of quasiperiodic regimes gradually decrease in size and disappear on the parameter plane period amplitude of external signal.

3 EXPERIMENTAL STUDYING

Now let us turn to experimental studies of two coupled Van der Pol oscillators driven pulsed action. A schematic of experimental system is shown in Fig.4. It include two self-generators: generator 1 (G1) and generator 2 (G2). Each generator consist of: oscillatory circuit, formed by inductance coils L_1, L_2 and capacitors C_1, C_2; nonlinear elements represented back-to-parallel semiconductor diodes (D_{1-6}), excitation of self-oscillations was carried out by negative resistance ($-r$), assembled on the basis of operational amplifier (OA4 and OA7). The coupling between generators was provided by variable resistor R_C, included between identical points of generators. Generator 1 was driven by external force represented short rectangular pulses of positive polarity formed standard generator G5-54. Operational amplifier OA1 was used to bypassing pulses generator G5-54 and scheme. Amplifiers OA2 and OA3 was used to bypassing measuring devices. Amplifiers OA5 and OA6 was used to differentiation the output signals of self-generators.

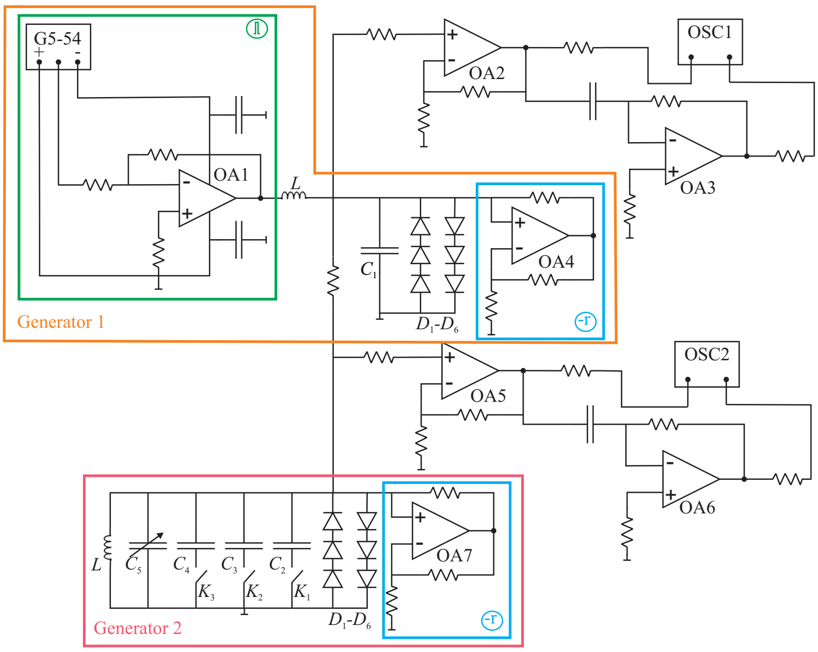

Figure 4: Scheme of experimental circuit for two coupled Van der Pol oscillators driven pulsed force.

Switches K_1 - K_3 and capacitors C_2 - C_4 were used to hard (stepwise) frequencies tuning of generator 2 and capacitor of variable capacity C_5 was used to smooth frequencies detuning. The parameters of corresponding elements of generators (inductance of oscillator circuits, parameters of diodes, elements of negative resistance, differential amplifiers) were closer.

In Fig. 5 shows reduced schema of experimental circuit. Writing Kirchhoffs law for points, marked U and $\tilde{U}$ in Fig. 5, and supposing that diode

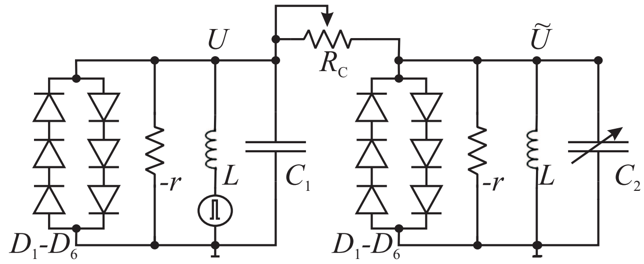

Figure 5: Reduced scheme of experimental circuit shown in Fig. 4.

characteristics described by the equation: $f(U) = I_0(e^{\frac{U}{\varphi}} - 1)$, where I_0 is saturation current of diode, $\varphi = \frac{kT}{e}$ is temperature voltage, after some algebraic transformations we can lead to:

$$\ddot{U} - \frac{1}{C_1}((\frac{1}{r} - \frac{2I_0}{3\varphi}) - \frac{I_0}{27\varphi^3}U^2)\dot{U} + \frac{1}{C_1 L}U + \frac{1}{R_C C_1}(\dot{U} - \dot{\tilde{U}}) = A\sum\delta(t - nT),$$

$$\frac{C_2}{C1}\ddot{\tilde{U}} - \frac{1}{C_1}((\frac{1}{r} - \frac{2I_0}{3\varphi}) - \frac{I_0}{27\varphi^3}\tilde{U}^2)\dot{\tilde{U}} + \frac{1}{C_1 L}\tilde{U} + \frac{1}{R_C C_1}(\dot{\tilde{U}} - \dot{U}) = 0, \tag{2}$$

where $A = \frac{V_0}{C_1 L}$ is amplitude of pulsed signal. Thus we get the equations similar Eqs. (1) but in dimensional form.

In the experiment we could change coefficient of coupling oscillators, varying resistance R_C , frequency detuning, varying capacity of capacitors. Also we could change the parameters of external signal: rms voltage, applied by pulses generator V, from which we could calculate voltage, applied to the generator 1 (amplitude of external signal) $V_0 = V\frac{\tau}{T}$; period of pulses T and pulse length τ, which was chosen enough small respect period of pulses, $\tau = 150mks$.

Let us investigate autonomous system of two coupled Van der Pol oscillators without external action. In the experiment the main parameters was: $r = -410kOm$, $C_1 = 20000nF$, $L = 0.947Hn$, common resistance R_C varying from 1 to 50 kOm, and capacitance C_2 varying from 1.5 nF to 4 nF. As a frequency detuning parameter was used $\Delta = lg(\frac{C_1}{C_2})$ and as a parameter of coupling was used $\mu \approx \frac{1}{R_C}$. The experimentally chart of dynamical regimes on the parameter plane frequency detuning - coefficient of coupling is given in Fig. 6, here you can see area of enough large frequency detuning of oscillators.

This chart contain the same character areas, that you can find in analytical and numerical studying [1, 4]: area of synchronization 1:1, area of

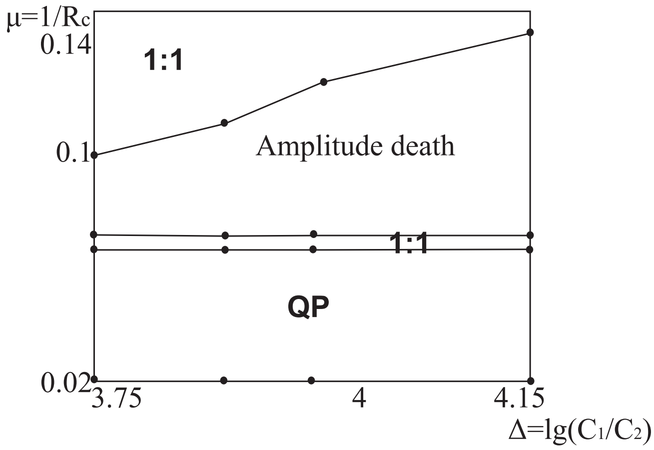

Figure 6: Experimental chart of dynamical regimes for system of two coupled self-generators without external force.

quasiperiodic regimes and area of amplitude death. One of the differences is narrow band of synchronization 1:1 between area of amplitude death and area of quasiperiodic regimes. The presence of this band due to the effect of broadband synchronization in the system of two dissipativelly coupled nonidentical Van der Pol oscillators [10]. Though the oscillators were assumed to be identical by control parameter (parameter λ), but the realization of condition of identicality in experiment is enough difficult problem. In Eqs. (2) the parameter responsible for frequency detuning $\Delta = \frac{C_1}{C_2}$ stand before the second derivative. If the capacities of generators are different, it is necessary to introduce frequency detuning for manifesting of amplitude death effect, then automatically to the second term of Eqs. (2) nonidenticality is added. Thus, we get two nonidentical by negative resistances generators. That was why the parameters of negative resistances were choose so that the band of broadband synchronization by coefficient of coupling was as narrow as possible.

Let us fix parameter of frequency detuning, i.e. capacity of capacitors: $C_1 = 20000nF$, $C_2 = 3nF$, choose the value of coupling parameter R_C, so that autonomous system demonstrate synchronous self-oscillations ($R_C = 8.5kOm$), and add external force. The experimental chart of dynamical regimes on the parameter plane (T, V_0) for mentioned parameters is given in. Fig. 7a.

Fig. 7a we should compare with Fig. 1a. As seen from comparison,

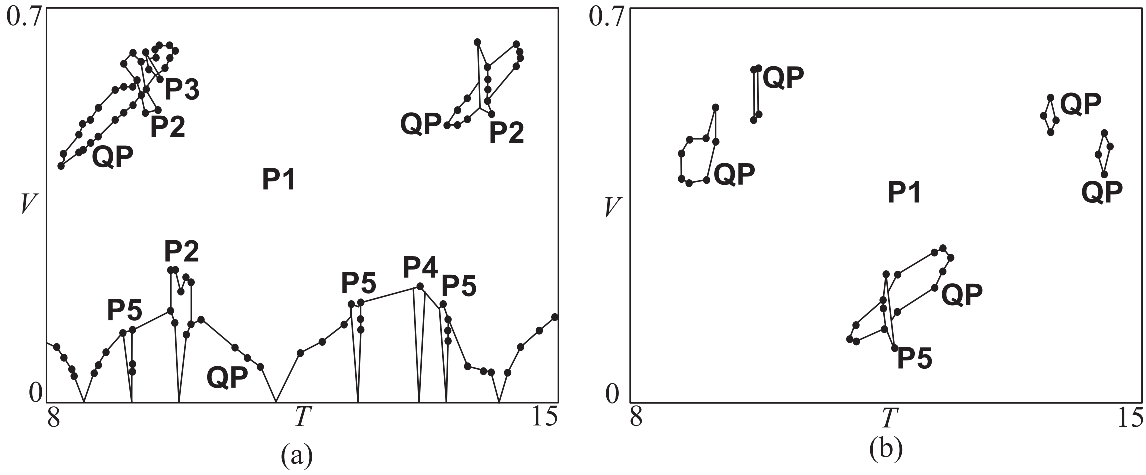

Figure 7: Experimental charts of dynamical regimes for system of two coupled self-generators driven pulsed force: a) autonomous system demonstrates regime of synchronization $R_C = 8.5kOm$; b) autonomous system demonstrates regime of amplitude death $R_C = 10kOm$.

the main features of synchronization picture for experimental system is keep. At the small value of amplitude of external signal we can observe the set of synchronization tongues embedded in the area of quasiperiodic regimes. Also the subharmonic tongues of synchronization, for example tongues of period-4 and period-5 and large tongue of period-2, are observed. At increasing amplitude of external force on the parameter plane the regimes of period 1 is dominated, but there are several islands of quasiperiodic regimes.

Now let us increase the value of the coupling parameter R_C so that we turn to the area of amplitude death. Experimental chart of dynamical regimes on the parameter plane of external force for this case is given in Fig.7b. Let us compare Fig. 7b and Fig.1b. As seen from comparison, the transition of autonomous system to the area of amplitude death is accompanied by occurrence of threshold of initializing of quasiperiodic regimes. The areas of quasiperiodic regimes form isolated islands on the parameter plane of external force. The narrow tongues of higher order intersect the islands of quasiperiodic regimes. The islands, which we observe in Fig. 7a, decrease in size.

Examples of phase portraits for the system of two coupled Van der Pol self-generator under periodic pulsed action photographed from the oscilloscope screen is given in Fig.8. In left column there are the phase portraits

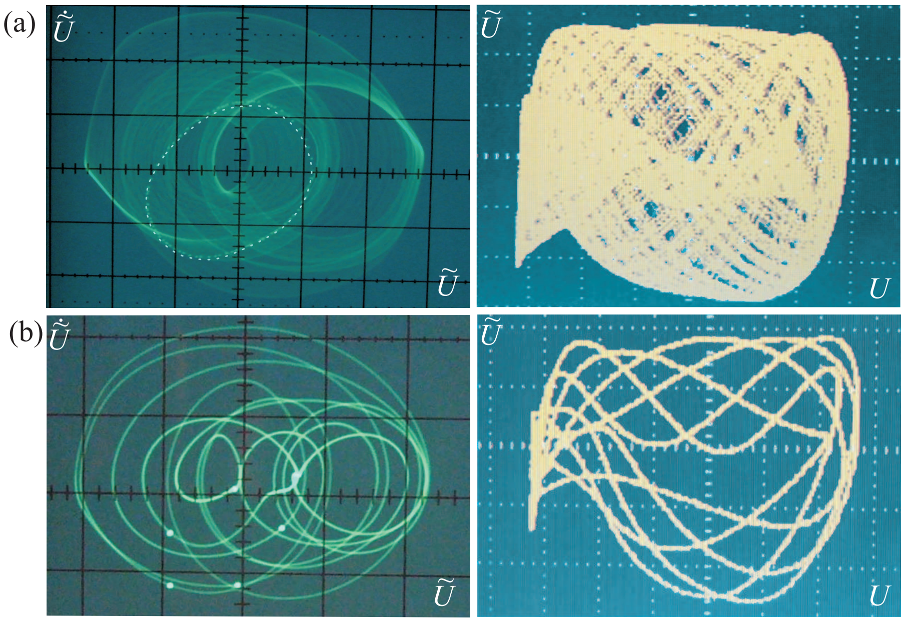

Figure 8: Projections of phase portraits for the system of two coupled self-generator driven pulsed force photographed from the oscilloscope screen: a) quasiperiodic regime; b) regular regime of period-5.

on the variable plane $(\tilde{U}, \dot{\tilde{U}})$ of generator 2, bright points mark stroboscopic section of Poincar. In experiment the section of Poincar is visualized in the next way: the pulse inputs to "Z", controlling brightness of electronic beam of oscilloscope. As a result, on the screen a picture corresponding to attractor in stroboscopic section was formed. Introduction of time shift between acting and brightening pulses allowed to displace section of attractor in the projection. In right column the Lissajous figures $(U, \tilde{U})$ are showed. In phase portrait in Fig. 8a we can see closed invariant curve in the stroboscopic section of Poincar, it proves that realizing regime is quasiperiodic. Fig. 8b correspond regular regime of period-5, it is clear illustrated by number of fixed points in the stroboscopic section of Poincare. There is some feature: phase trajectories are enough complex orbits containing a large number of turns despite its small period in the section of Poincare. It illustrate complex character of moving of coupled oscillators in the regime when dissipative coupling suppresses self-oscillations.

At further increasing of coupling parameter R_C, island of quasiperiodic regimes disappear. For example, at $R_C = 12kOm$ coupled oscillators driven pulsed signal demonstrate only regime of synchronization 1:1 (period-1 in the section of Poincar).

4 CONCLUSION

The effect of amplitude death in dissipatively coupled self-oscillatory oscillators is character general effect. As the radiophysics experiment and numerical simulations are showed the external force for system in this regime can initiate quasiperiodic oscillations. In this case on the parameter plane of external force occur islands of quasiperiodic regimes which are intersected by the set of small-scale areas of periodic regimes. The occurrence of quasiperiodic regimes has threshold character by amplitude of force. At further advance to the area of amplitude death in autonomous system, islands gradually disappear. Phase trajectories of oscillators for such regimes is enough complex orbits, containing a large number of turns despite its small period in the section of Poincar.

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