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# Hyperbolic chaos in systems with multiple delays: numerical test via angle criterion

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Hyperbolic chaotic attractors, like the Smale-Williams solenoid, manifest deterministic chaos justified in a rigorous mathematical sense [1]. Such attractors demonstrate strong stochastic properties and allow a detailed mathematical analysis. They are structurally stable with respect to variation of functions and parameters in the dynamical equations, and demonstrate insensitivity of chaos characteristics to noises, interferences etc. The hyperbolic attractors were commonly regarded as purified abstract mathematical images of chaos rather than something intrinsic to real world systems. However, recently many physically realizable systems with hyperbolic attractors have been offered [2].

An important class among nonlinear systems with complex dynamics is formed by systems containing time-delay feedback loops. An adequate mathematical description for these objects is based on differential equations with delays. Several examples of them were suggested as realizable devices for generation of rough hyperbolic chaos [3].

Recently we have developed a numerical test of hyperbolicity of chaotic dynamics in systems with a single delay [4]. The test is based on the angle criterion and includes computation of angle distributions between expanding, contracting and neutral manifolds of trajectories on the attractor [5]. In the present work we generalize this approach to systems with multiple delays and apply it to confirm the hyperbolicity of several previously reported time-delay chaotic systems.

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