Reconstruction of model equations to the problem of the body of elliptic cross-section falling in a viscous fluid

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By processing time series obtained from numerical solution of the plane problem for the motion of a body of elliptic cross-section with gravity in incompressible viscous fluid, a system of ordinary differential equations is reconstructed for approximate description of the dynamics. The postulated equations take into account the added masses, the force caused by the circulation of the velocity field, and the movement resistance forces, and the coefficients in these equations are evaluated using the least squares method to fit the observable time series data. Correspondence is illustrated of the finite-dimensional description and simulation based on the Navier – Stokes equations by portraits of attractors in regular and chaotic regimes. Moreover, the obtained coefficients provide a glimpse of the real contribution of various effects in the body dynamics.

The model equations in dimensionless variables are of the form

$$\begin{gathered} A\dot{u} = Bvw - Dvw - C|u|u - \sin\theta, \quad B\dot{v} = -Auw + Duw - E|v|v - \cos\theta, \\ \dot{w} = -Guv - H|w|w, \quad \dot{\theta} = w, \end{gathered} \tag{1}$$

$$\dot{X} = u\cos\theta - v\sin\theta, \;\; \dot{Y} = u\sin\theta + v\cos\theta. \tag{2}$$

The coefficients obtained by processing the results of the numerical solution of two-dimensional problem of the fall of the body of elliptic profile for the semi-axes a=0.486 cm and $b = a$/6=0.081 cm, viscosity η=0.001 Pa×s, fluid density ρ_f=1000 kg / m^3 are listed in the Table for variants with different densities of the body ρ_s.

ρ_s, kg/m^3	1710	2000	2300	2600	2900	3600
A	1.3945	1.3392	1.2581	1.1975	1.1551	1.0388
B	4.7378	3.9290	3.2845	2.7320	2.3245	1.9196
C	0.1069	0.0891	0.0873	0.0850	0.1209	0.1044
D	1.9730	1.8751	1.7952	1.6617	1.6221	1.2957
E	1.7720	1.5770	1.3803	1.3248	1.1254	0.7034
G	0.8681	0.8665	0.8516	0.7893	0.7710	0.5636
H	0.4130	0.3884	0.3163	0.2723	0.2775	0.0073

Figure 1 compares the trajectories of the fall resulting from a two-dimensional numerical solution of the problem with Navier – Stokes equations and within the finite-dimensional model (1), (2). Figure 2 compares portraits of attractors in the projection on the plane of the variables for the same modes.

Thus, in this report we have demonstrated a possibility of rather satisfactory approximate description of the motion of the body of elliptical cross-section under gravity in a fluid using ordinary differential equations reconstructed on the basis of the processing data from the numerical solution of the problem with the Navier – Stokes equations. The proposed approach is interesting, in particular, in relation to the control problems concerning motions of bodies in fluid as the description is much easier than the rigorous computations while the degree of quantitative compliance is better than that in the previously discussed phenomenological models.

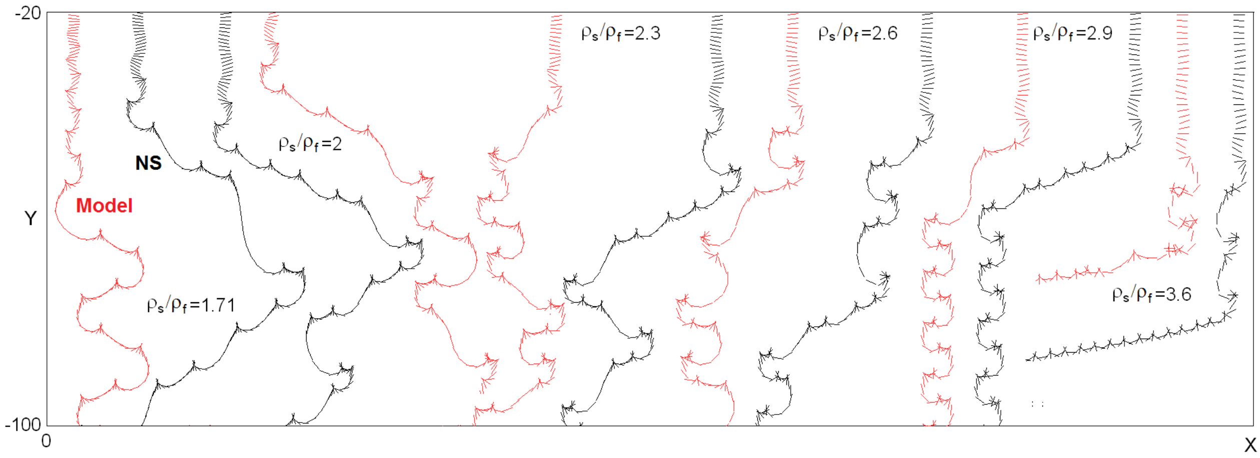

Fig. 1. Stroboscopic visualization of the body falling in a fluid: instant positions of the major axis of the ellipse at successive time points are shown based on the results of numerical simulation with the Navier – Stokes (NS) and obtained for the model (1), (2) with coefficients from the Table.

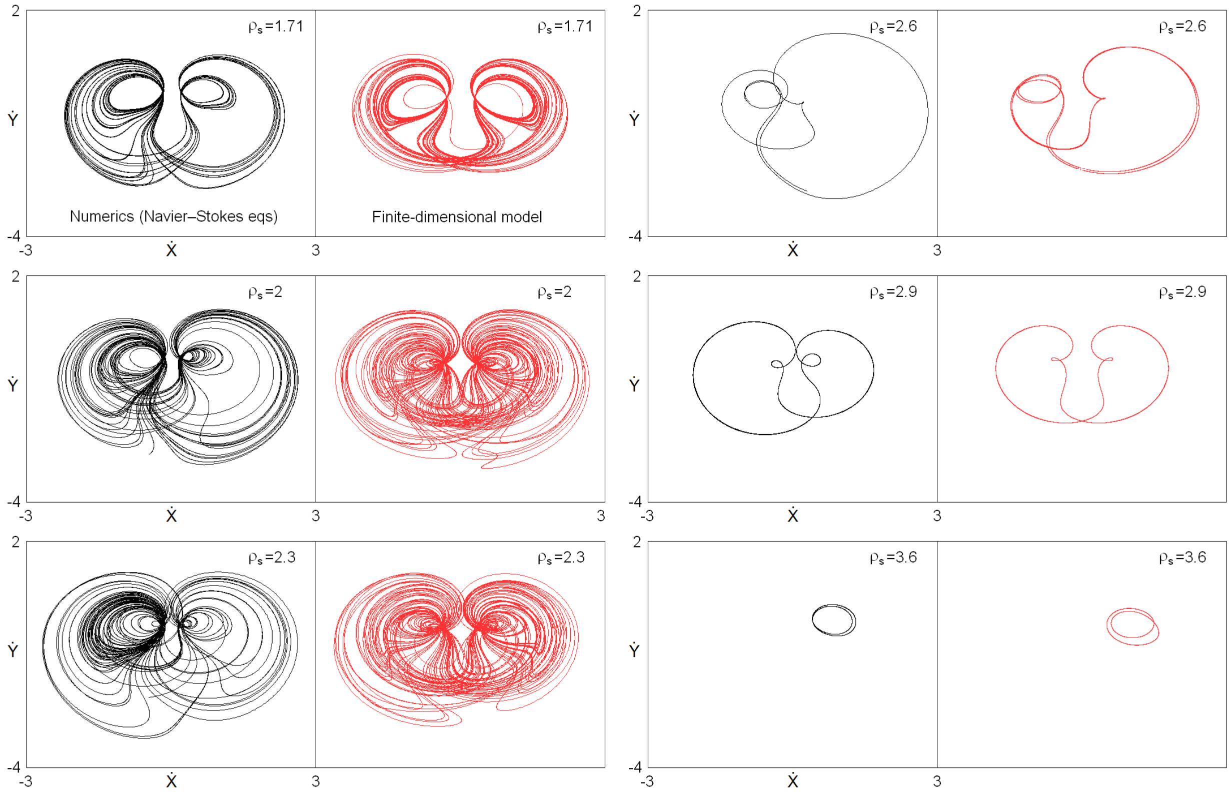

Fig. 2. Portraits of attractors in projection onto a plane $(\dot{X},\ \dot{Y})$ from numerical simulation of the dynamics with the Navier – Stokes equations (left columns) and from the model (1), (2) (right columns) for different body densities.

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