DESIGN PRINCIPLES AND ILLUSTRATIONS OF HYPERBOLIC CHAOS IN MECHANICAL AND ELECTRONIC SYSTEMS

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Abstract. The report contains an overview of approaches to constructing systems of electronic and mechanical nature manifesting the hyperbolic chaos.

Hyperbolic chaos has fundamental advantages for applications. First, it is characterized by roughness, or structural stability, i.e. insensitivity of chaos to slight variations of parameters and to a form of the evolution operator. Second, presence of a well-elaborated mathematical theory is attractive for developing profound opportunities for applications, for example, in communication systems. Until recently, however, hyperbolic chaos was represented exclusively by mathematical constructions (Smale-Williams attractor, Plykin attractor, and Anosov dynamics). If mathematicians develop their examples using geometric, topological, algebraic methods, a physicist for design of mechanical or electronic models with hyperbolic chaos should use another toolbox: oscillators, particles, interactions, feedback loops etc. [1].

The simplest approach is to consider a mechanical system, for example, a particle in a plane or in space, under periodic kicks, the magnitude and direction of which depend on the instantaneous position of the particle, implementing such conditions that the mapping over the period would correspond to some mathematically constructed map with a hyperbolic attractor [2].

The second approach is based on a use of two or more oscillators, which pass the excitation each other in such way that the complete cycle of the transfer corresponds to the expanding map for the angular variable, which corresponds to the phase of the oscillations [3]. A similar method may be applied to distributed systems, where the expanding map is subjected to a phase of spatial patterns, such as Turing structures or parametrically excited standing waves on a nonlinear string [4].

Several approaches are based on the purposeful design of models in the form of differential equations with right parts of periodically switched form. In particular, this made it possible to indicate an example of a system where an attractor of the Plykin type is realized [5].

One more recently developed direction is constructing mechanical systems starting from Anosov's dynamics on a surface of negative curvature. In particular, such dynamics are realized in the Thurston-Wicks-McKay-Hunt triple-linkage hinge mechanism. Several versions have been considered, where the mechanical constraint through hinges and pivots is replaced by potential interaction for the three rotators [6]. Introducing dissipation and feedback makes the system with hyperbolic chaos self-oscillatory. The Anosov type dynamics occur on an attractive set in the phase space but the attractor retains the hyperbolic nature due to the inherent structural stability. Been inspired by the mechanical system, it appears possible to implement an electronic circuit operating as a hyperbolic chaos generator, where a phase-locked loop is used as an analog of the mechanical rotator.

All the mentioned principles of design of systems with hyperbolic chaos are illustrated by concrete examples represented by differential equations or electronic circuits, and results of simulation of their functioning are presented.

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